Finding Communities

Social Networks Analysis and Graph Algorithms

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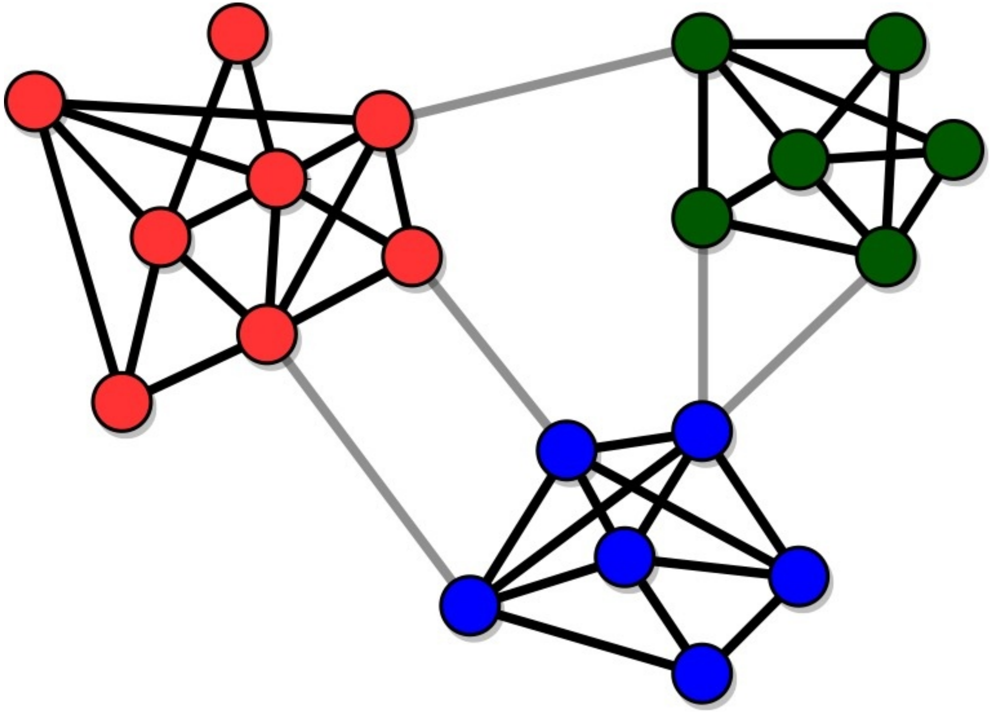
Sources

- A. L. Barabási (2016). Network Science – Chapter 09
- D. Easly and J. Kleinberg (2010). Networks, Crowds, and Markets – Chapter 03
- F. Menczer, S. Fortunato, C. A. Davis (2020). A First Course in Network Science – Chapter 06
- URLs cited in the footer of slides

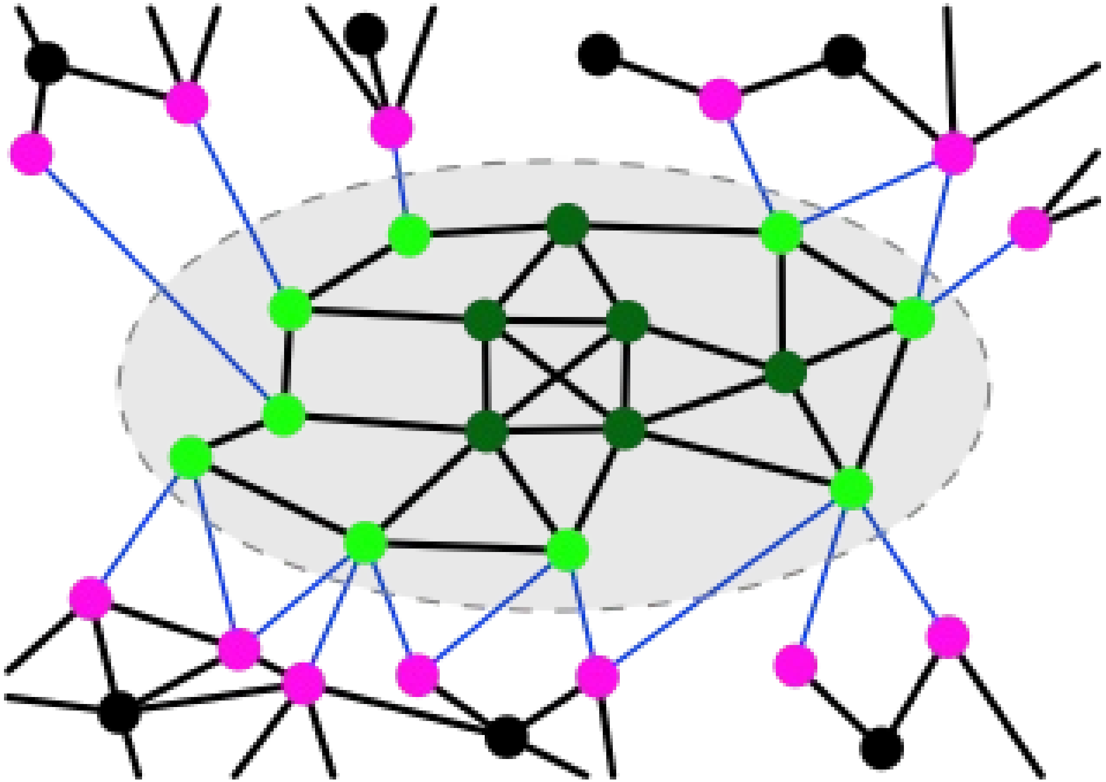
Example with clear community structure



Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Characterizing one community

Communities are connected and dense



Given a community C

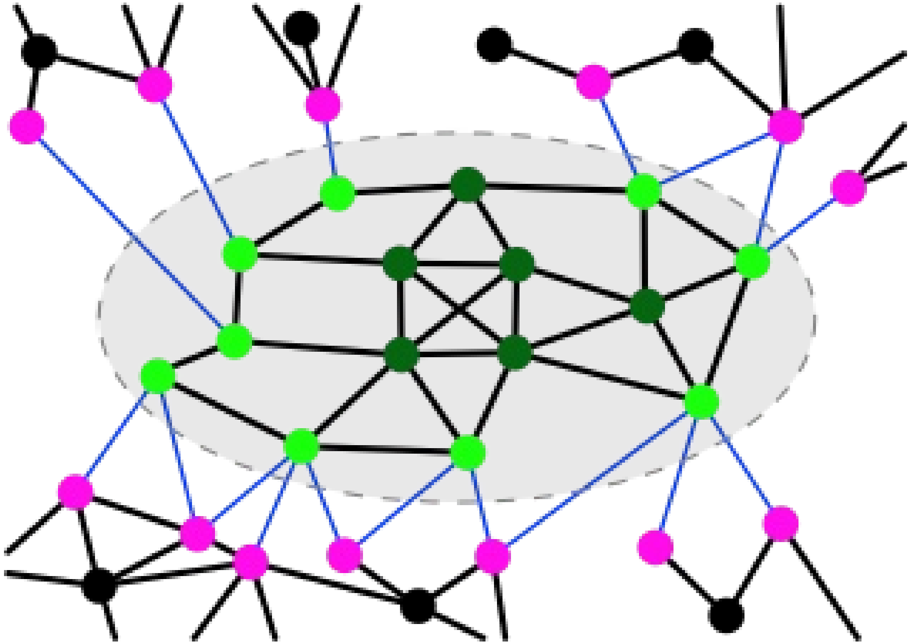
Internal degree $k^{int}(C)$ considers only nodes inside the community

External degree $k^{ext}(C)$ considers only nodes outside the community

$$k_i = k_i^{\text{int}}(C) + k_i^{\text{ext}}(C)$$

Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Strong community



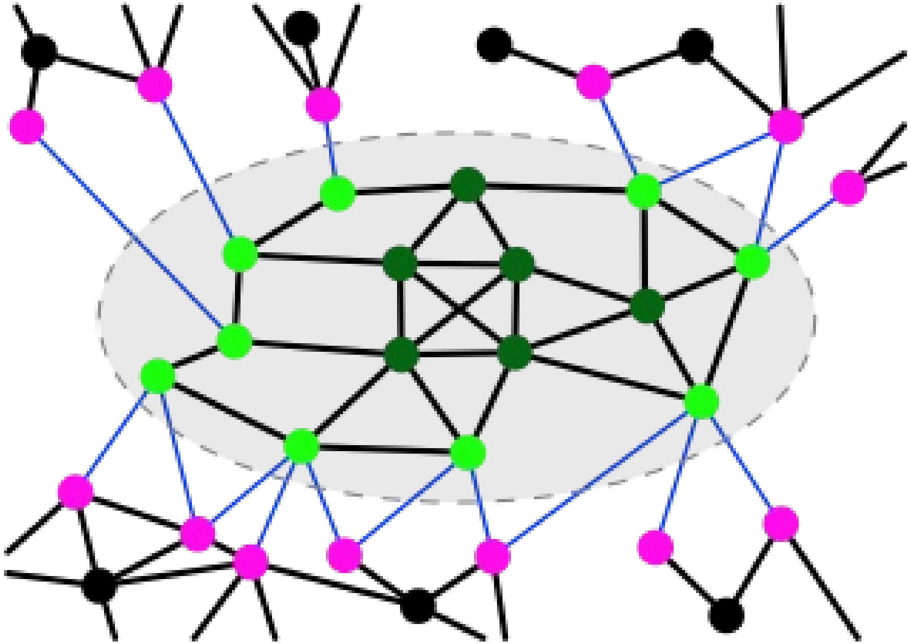
A community C is **strong** if **every node** ***i*** within the community satisfies:

$$k_i^{\mathrm{int}}(C) > k_i^{\mathrm{ext}}(C)$$

- Is the community of green nodes (dark green and light green) a strong community?
- What is the difference between dark green and light green nodes?

Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Weak community



A community C is **weak** if **on aggregate** nodes satisfy:

$$\sum_{i \in C} k_i^{\mathrm{int}}(C) > \sum_{i \in C} k_i^{\mathrm{ext}}(C)$$

- All communities satisfying the strong property satisfy the weak one

Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Exercise

Is community A strong, weak, both?

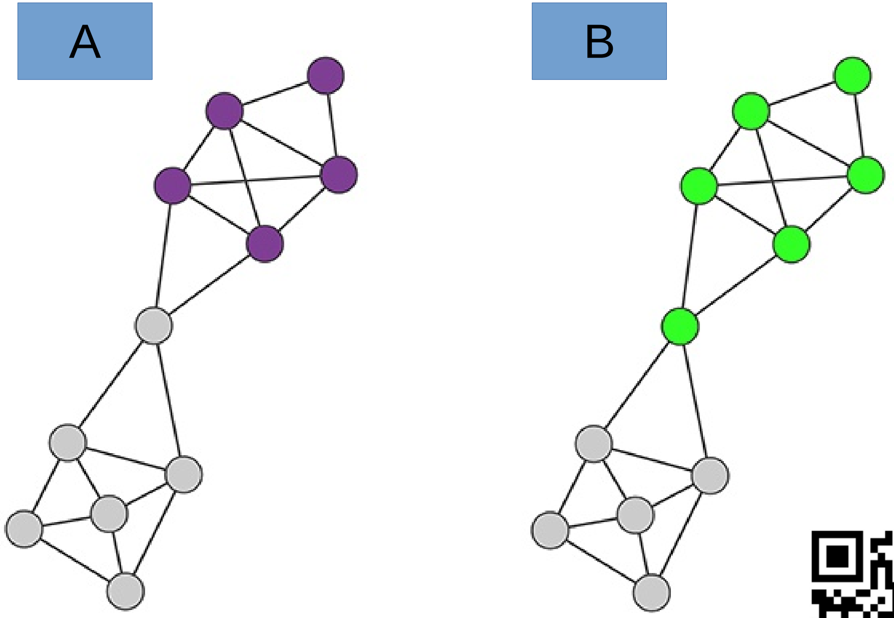
Is community B strong, weak, both?

A community C is **strong** if, for all nodes *i* within the community:

$$k_i^{\text{int}}(C) > k_i^{\text{ext}}(C)$$

A community C is **weak** if:

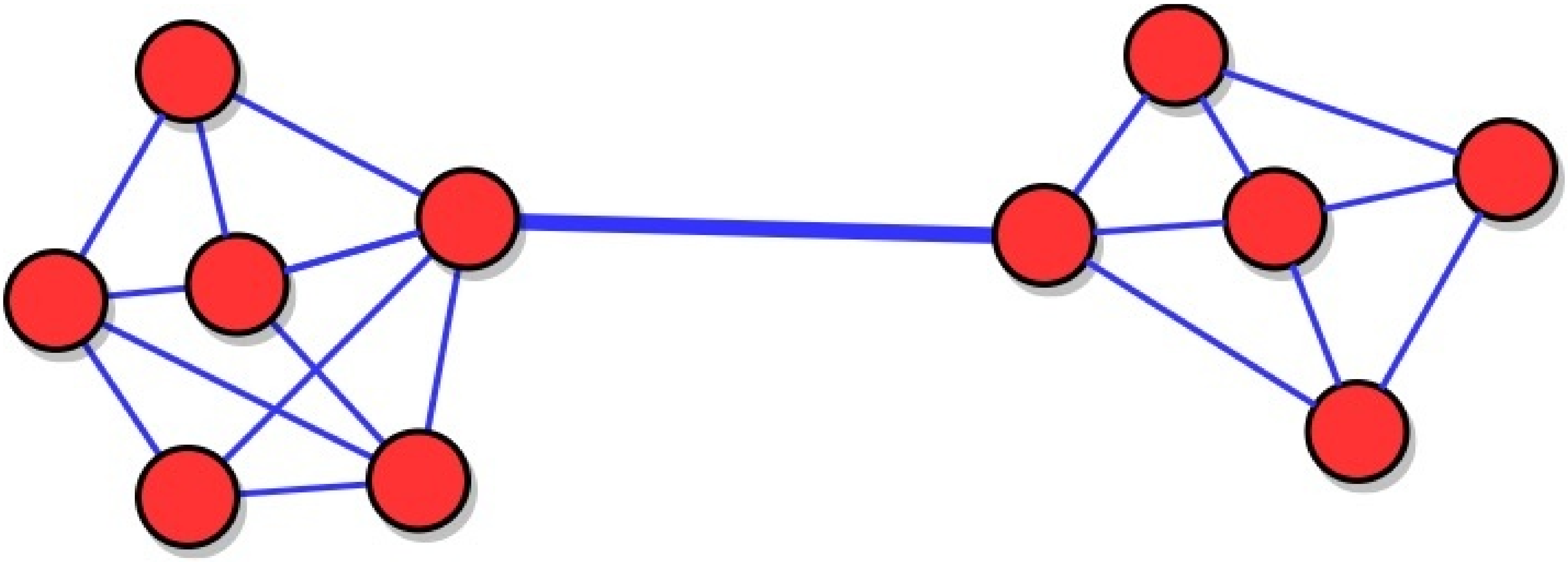
$$\sum_{i \in C} k_i^{\text{int}}(C) > \sum_{i \in C} k_i^{\text{ext}}(C)$$



Pin board: https://upfbarcelona.padlet.org/chato/tuxhfgcan59orweh

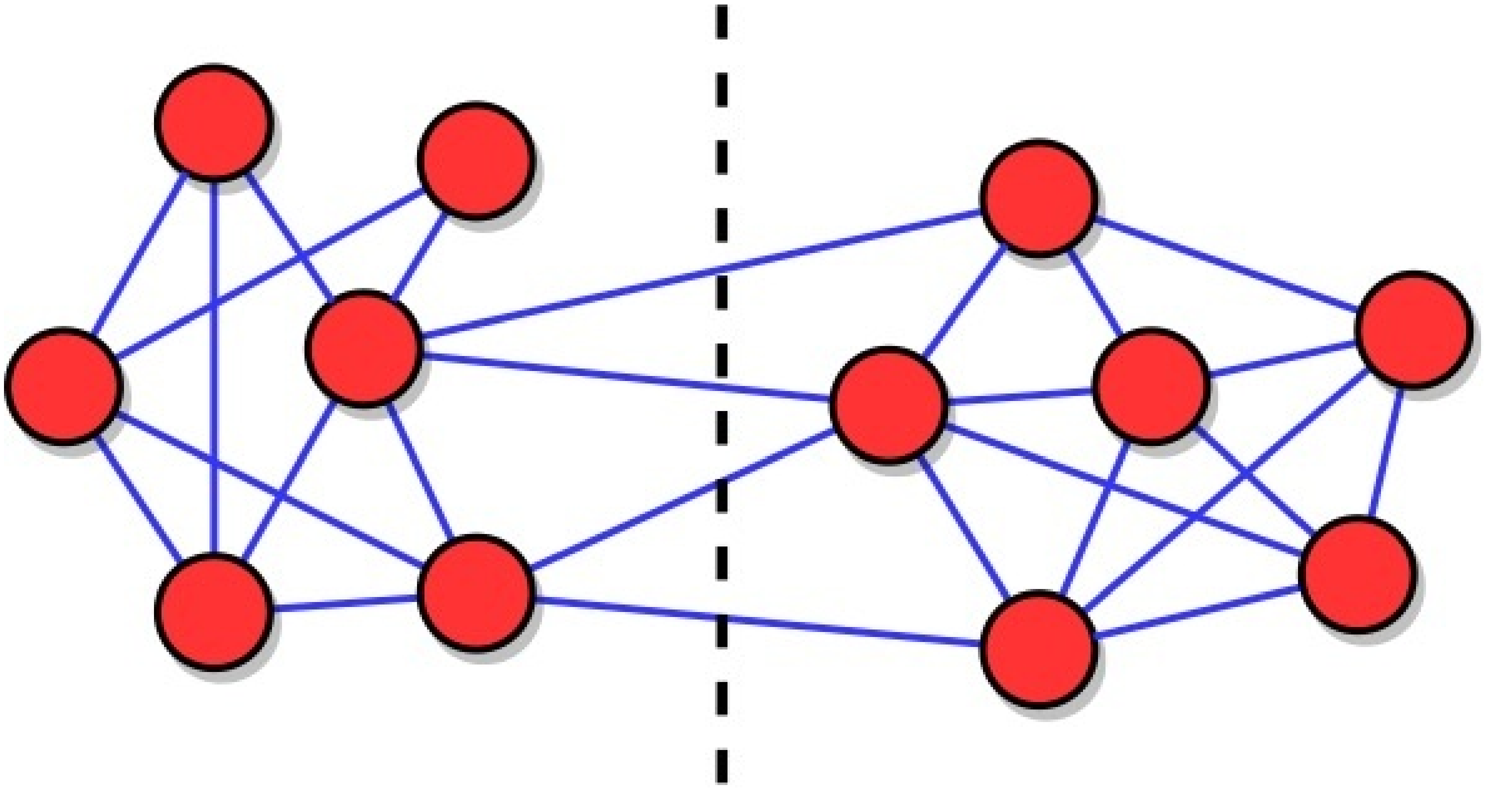
Finding two communities: network bisection

A graph that is easy to bisect



Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Graph bisection: finding a minimal “cut”

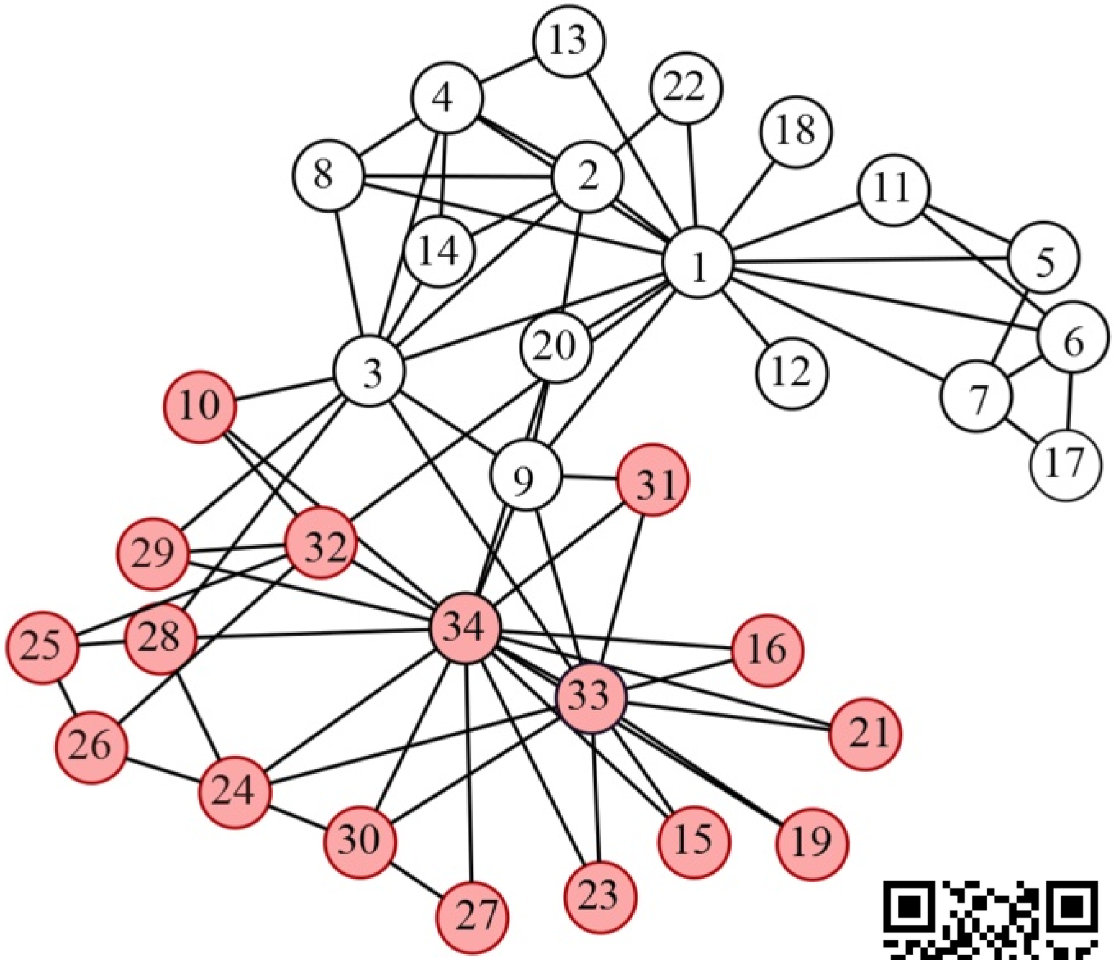


Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Simple exercise

Cut size under bisection

- What is the size of the white-red cut?
- If node 9 goes to the red component, what is the size of the white-red cut?



Pin board: https://upfbarcelona.padlet.org/chato/4qz0k8ro0zquen1

Finding multiple communities: a divisive method

Hierarchical graph partitioning

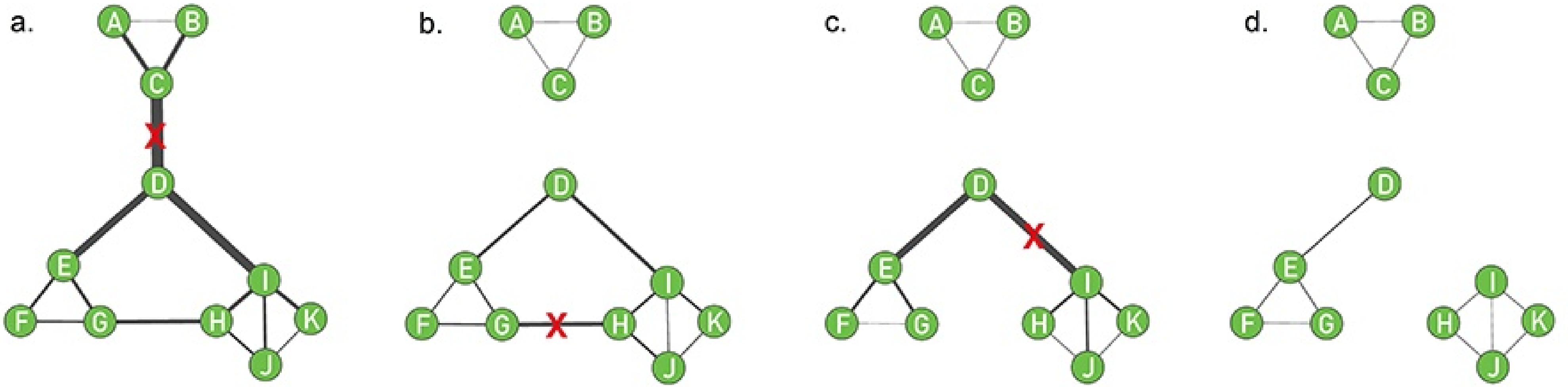
Until there are edges in the graph

Find an edge e that bridges two communities

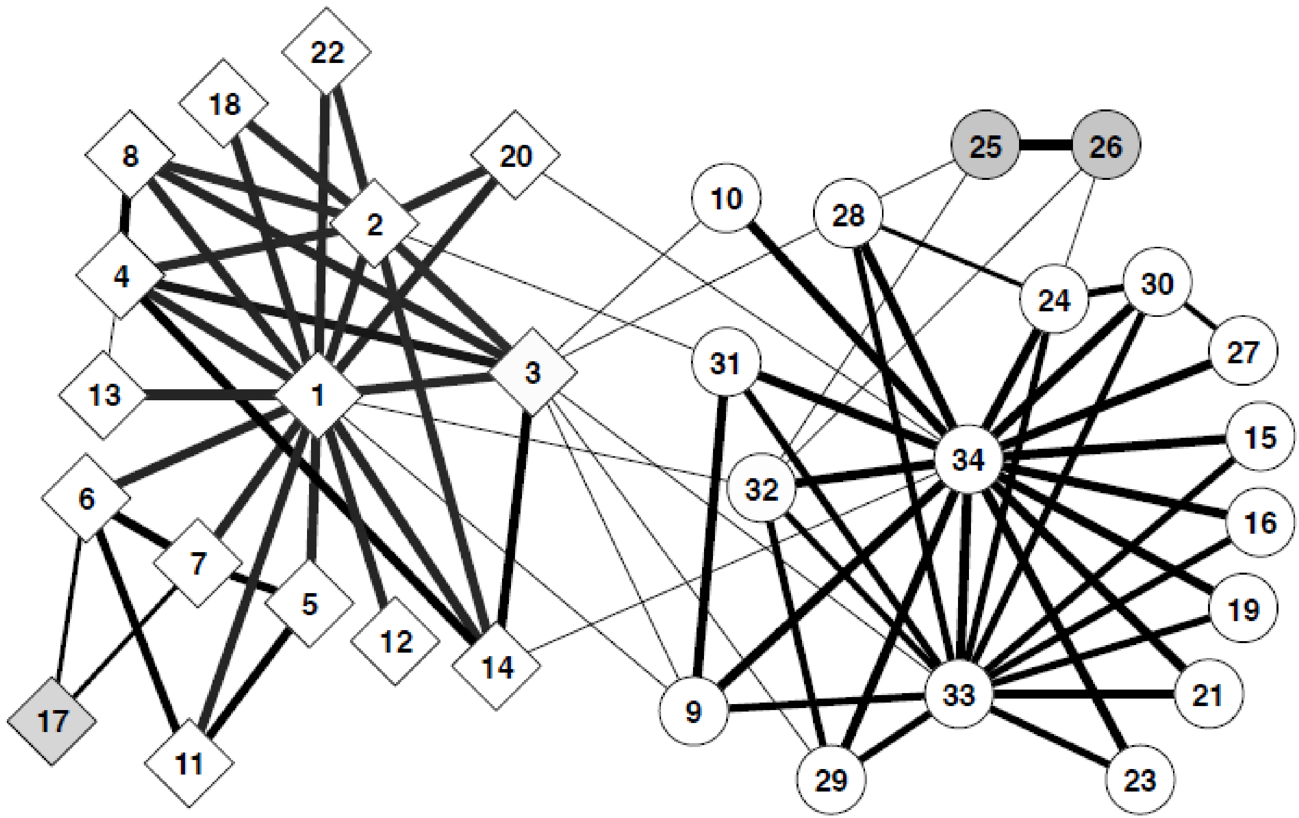
Remove edge e

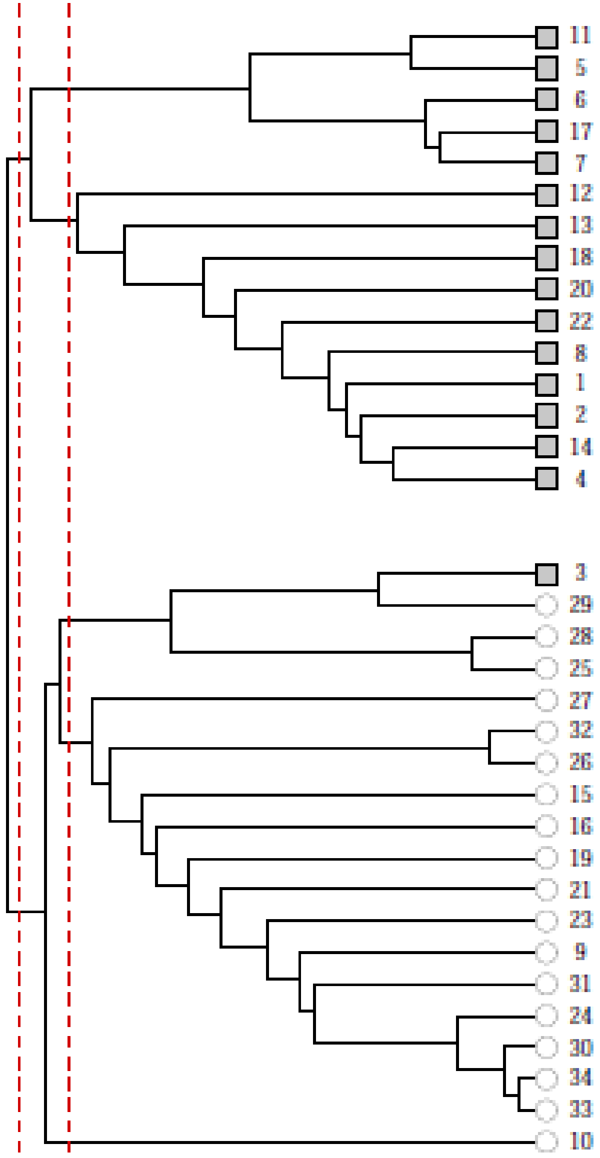
The Girvan-Newman algorithm

- Repeat:
  - Compute edge betweenness
  - Remove edge with larger betweenness



Example: Karate Club





https://slidetodoc.com/online-social-networks-and-media-community-detection-1/

Quantifying multiple communities: modularity

Measuring a partition in a graph

- **Modularity** (or one of its variants) is a popular method to determine how good a partition is on a graph
- It compares the observed number of internal links in each partition, against the expected number of internal links if those internal links had been placed at random

Modularity of a partition

$$Q = \frac{1}{L}\sum_{C}\left(L_C - \frac{k_C^2}{4L}\right)$$

- $L =$ number of links in the network
- $L_C =$ number of internal links in community C
- $k_C =$ sum of degree of nodes in C

Modularity of a partition (cont.)

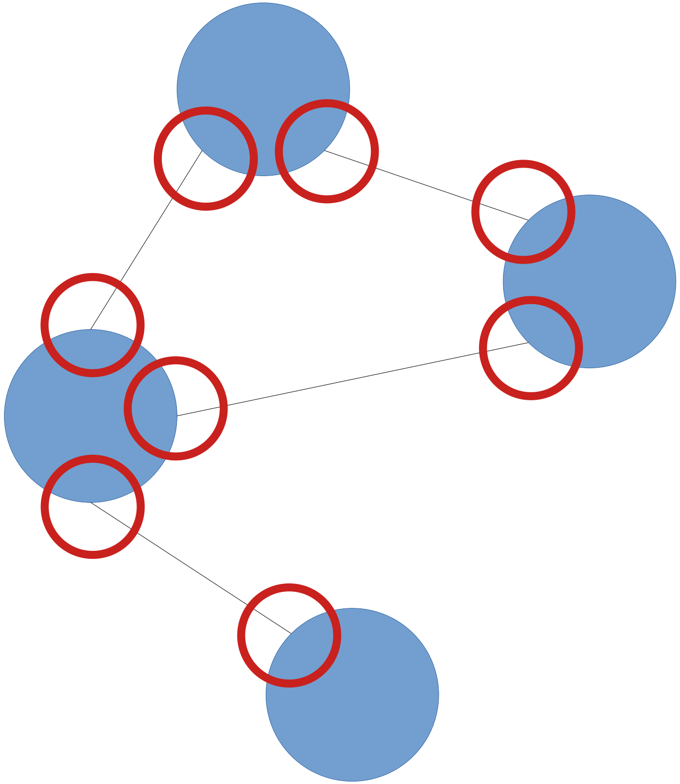
$$Q = \frac{1}{L} \sum_{C} \left(L_C - \frac{k_C^2}{4L} \right)$$

Expression in parenthesis is the difference between observed and expected internal links in community C

- L = number of links in the network
- L_C = number of internal links in community C
- k_C = sum of degree of nodes in C
- $k^2{}_C / 4L$ = **expected** number of internal links in community C

Where does $k_c^2/4L$ comes from?

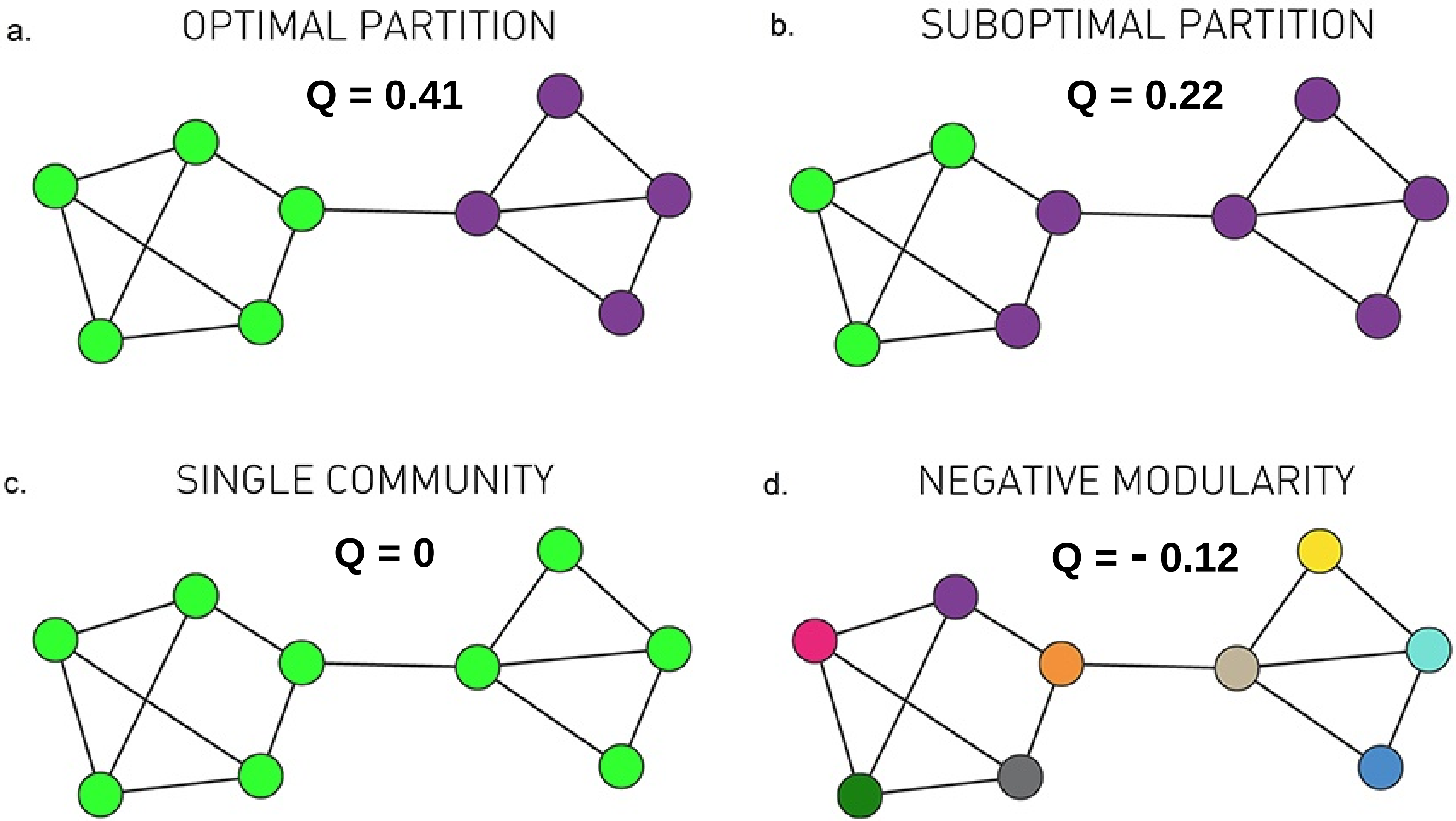
- A **link "stub"** is a connection between a link and a node
- There are $2L$ stubs in a network
- There are as many stubs as the sum of the degree of nodes



Modularity formula explained

$$Q = \frac{1}{L}\sum_C \left(L_C - \frac{k_C^2}{4L}\right)$$

- There are L_C internal links in C
- Total number of stubs in nodes in C is k_C
- Total number of stubs in the network is $2L$
- Probability of chosing two stubs in C: $(k_C/2L)^2 = k_C^2/4L^2$
- The **expected number** of links joining two stubs in C is $L(k_C^2/4L^2) = k_C^2/4L$
- The **observed number** is L_c
- Q has a range: $Q \in [-1, +1]$



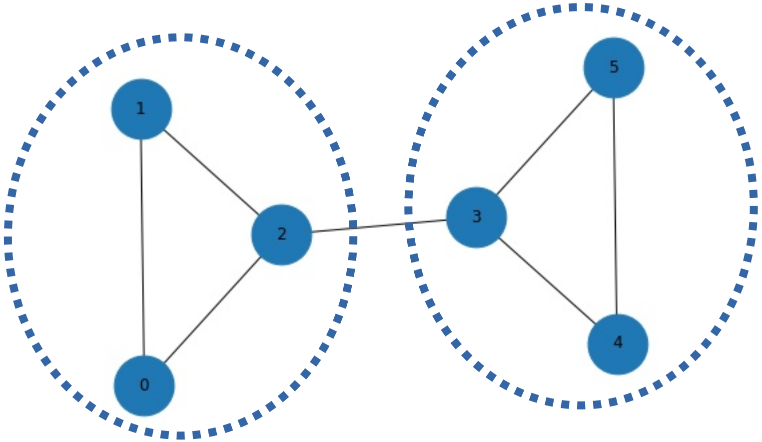
http://networksciencebook.com/chapter/9#modularity

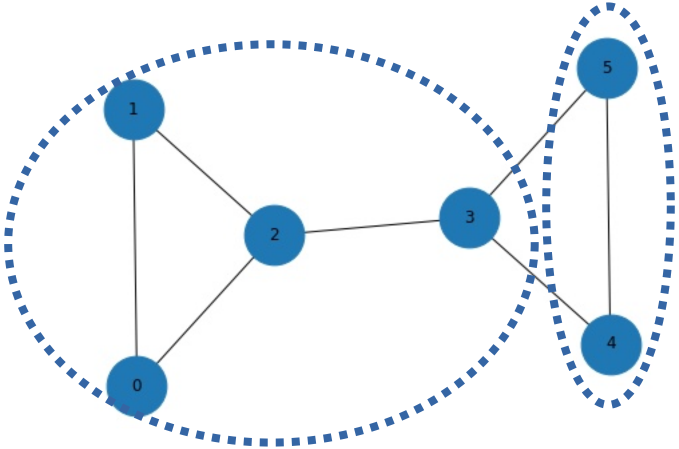
$$Q = \frac{1}{L}\sum_{C}\left(L_C - \frac{k_C^2}{4L}\right)$$

Exercise

- What is the modularity of the partition {0, 1, 2}, {3, 4, 5}?
- What is the modularity of the partition {0, 1, 2, 3}, {4, 5}?

$$Q = \frac{1}{L} \sum_{C} \left(L_C - \frac{k_C^2}{4L} \right)$$





Pin board: https://upfbarcelona.padlet.org/chato/lak2lnp9m3jc1naj

Summary

Things to remember

- Strong and weak community
- The concept of “cut” in graph bisection
- Girvan-Newman’s algorithm
- Modularity

Practice on your own

- Check the modularity computations in the example on the slide marked ★ : (a) optimal partitioning into two communities, (b) suboptimal partitioning into two communities, (c) all the nodes in a single community, (d) one community per node
- You can check your answers with networkx.algorithms.community.modularity