Preferential Attachment (BA Model)

Social Networks Analysis and Graph Algorithms

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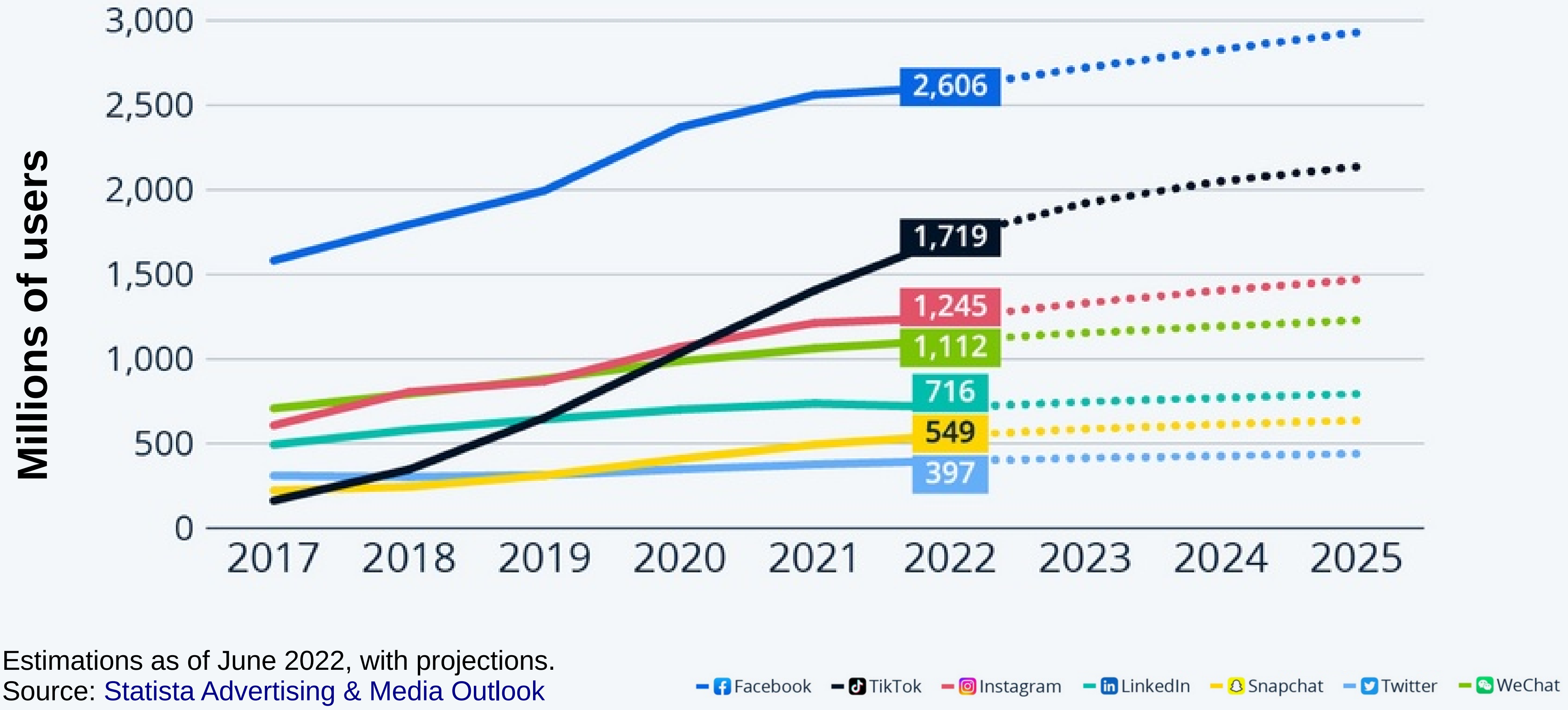
Contents

- The BA or preferential attachment model
- Degree distribution under the BA model
- Distance distribution under the BA model
- Clustering coefficient under the BA model

Sources

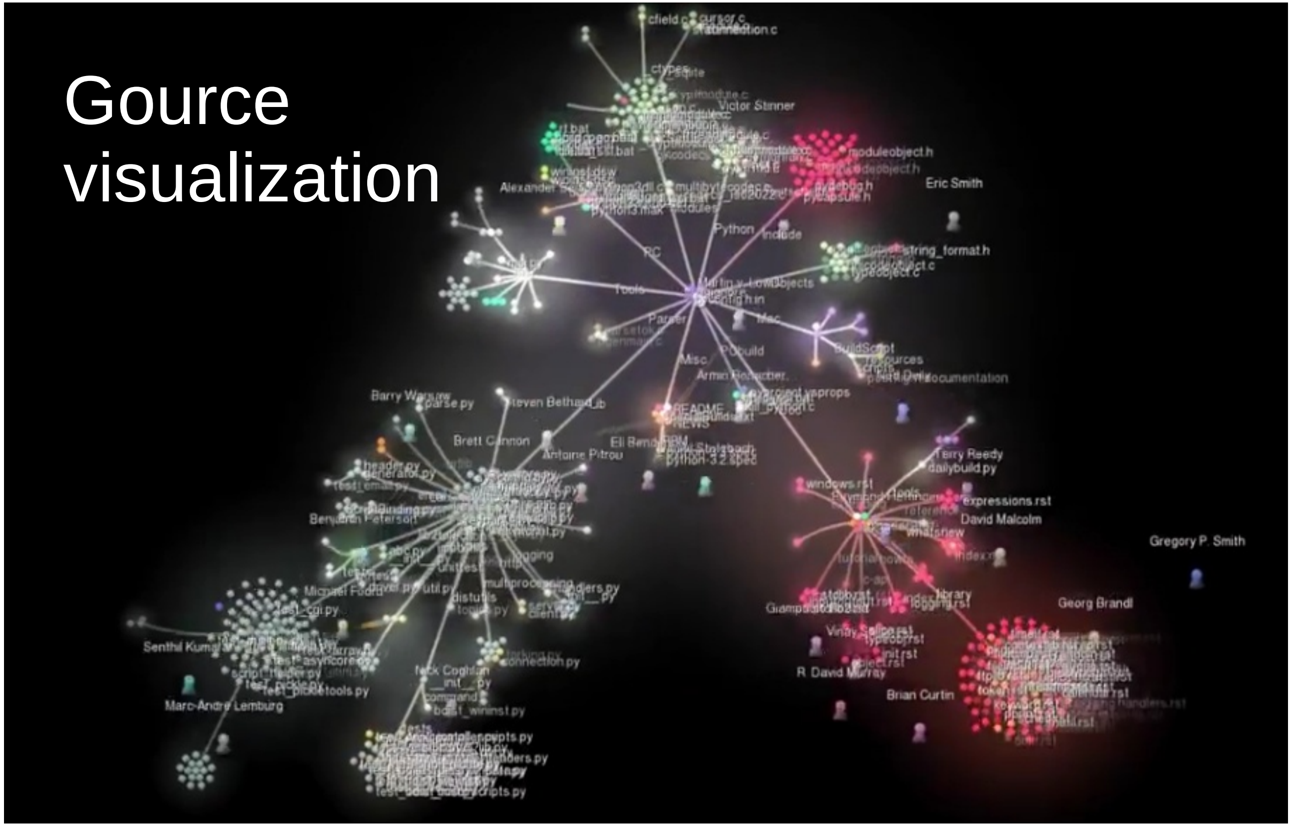
- A. L. Barabási (2016). Network Science – Chapter 05
- R. Srinivasan (2013). Complex Networks – Chapter 12
- D. Easley and J. Kleinberg (2010): Networks, Crowds, and Markets – Chapter 18
- Data-Driven Social Analytics course by Vicenç Gómez and Andreas Kaltenbrunner

Social networks grow over time



Estimations as of June 2022, with projections.
Source: Statista Advertising & Media Outlook

Growth of an Open Source Project: Python



https://www.youtube.com/watch?v=cNBtDstOTmA

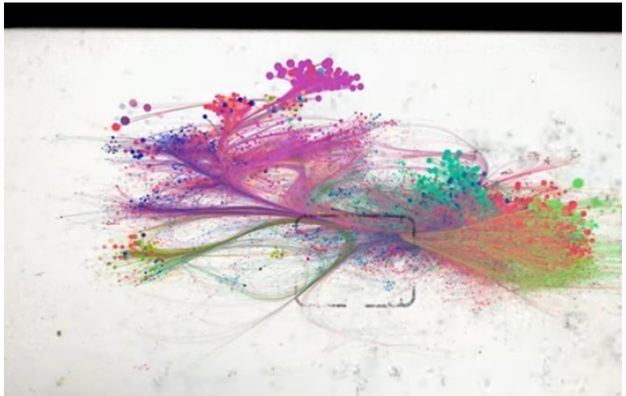
We have seen *what* but not *how*, or *why*

- Power-law degree distributions are prevalent
- We will give a possible answer to ***how***
- For now, we will not answer ***why***

Preferential Attachment

Video (04:43-06:45)
by Albert-László Barabási (cont.)





https://www.youtube.com/watch?v=RfgjHoVCZwU

From “Most real-networks do not form by connecting pre-existing ...”
To “… the same universal architecture.”

Growth

- Suppose there are two web pages on a topic, one with many inlinks the other with few, **which one am I most likely to link to?**
- Which **scientific papers are read?**
- Which **book authors sell more?**
- Which **actors are more sought after?**


BURNS
UNIVERSITY
Ut
Uberiores
Divites
Our motto: *Ut uberiores divites.*
FOX 32

Preferential attachment simulation

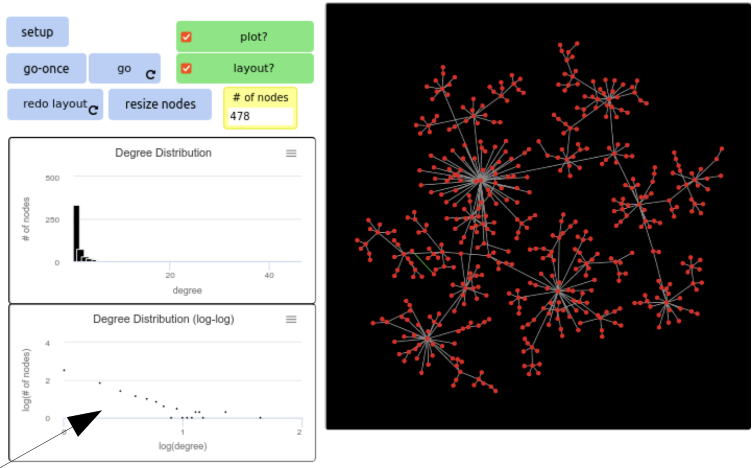
https://www.youtube.com/watch?v=4GDqJVtPEGg

Exercise

Slope of degree distribution

Go to netlogoweb.org/launch and select:
“Sample Models / Networks / Preferential Attachment”

- Execute in Netlogo Web the [“Preferential Attachment” program]:
  - Click “setup”
  - Click “go”
  - Let it run to ~500 nodes



- **Guess the slope of the degree distribution in log-log scale**

Pin board: https://upfbarcelona.padlet.org/chato/y8kw9jcjlluo2p8c

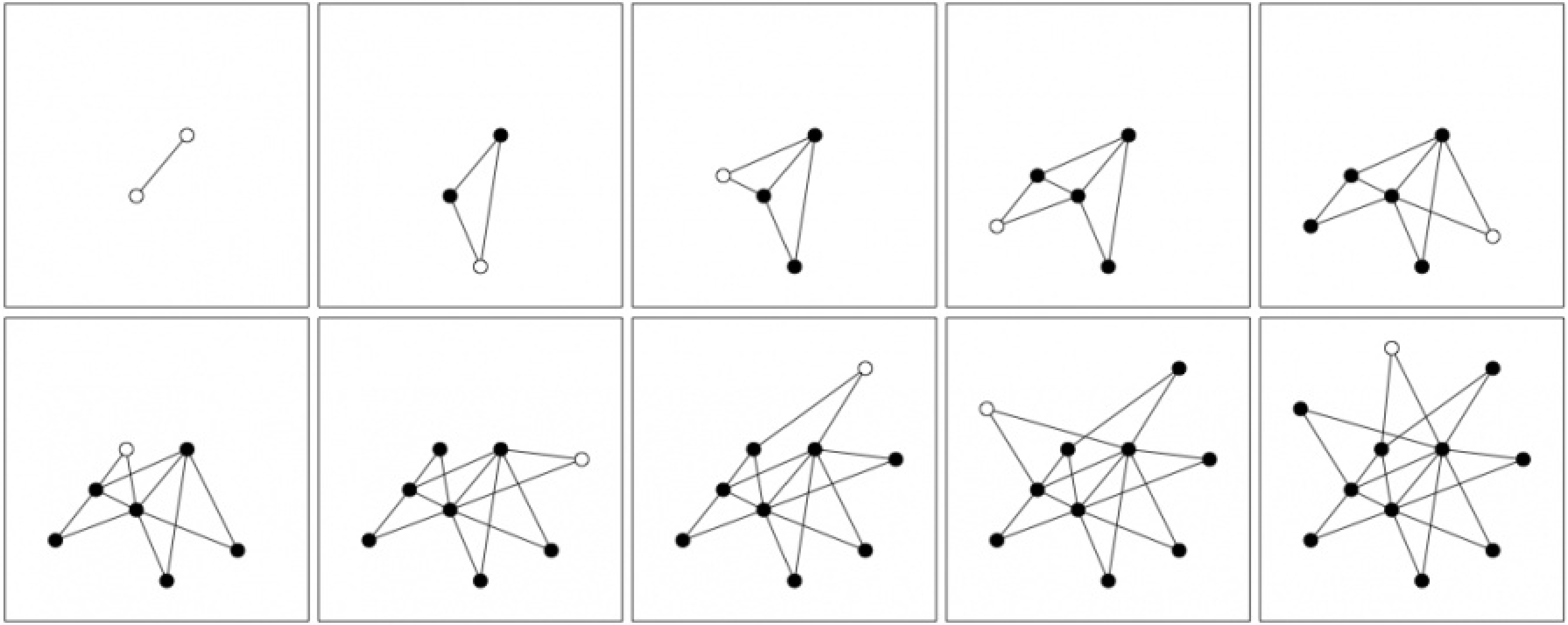
The Barabási-Albert (BA) model

- Network starts with m_0 nodes connected arbitrarily as long as their degree is ≥ 1
- At every time step we add 1 node
- This node will have $m \leq m_0$ outlinks
- The probability of an existing node of degree k_i to gain one such link is

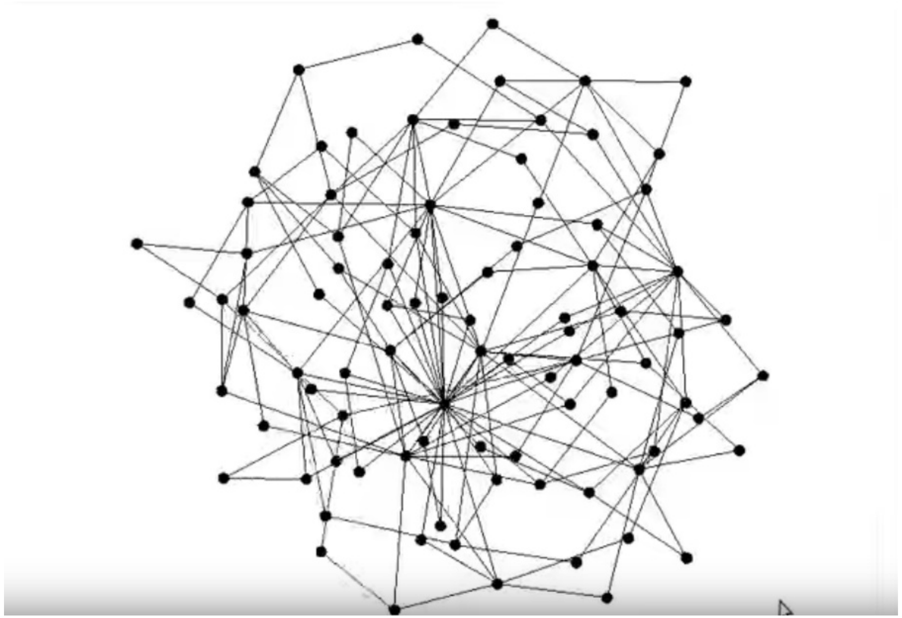
$$\Pi(k_i) = \frac{k_i}{\sum_{j=1}^{N-1} k_j}$$

In an ER network, $\Pi(k_i) = \frac{1}{N-1}$

Example ($m_0 = 2; m=2$)



Network growth with $m=2$



https://www.youtube.com/watch?v=wocaGeNKn7Y

The Barabási-Albert (BA) model

- Network starts with m_0 nodes connected arbitrarily as long as their degree is ≥ 1
- At every time step we add 1 node
- This node will have m outlinks $(m \leq m_0)$
- The probability of an existing node of degree k_i to gain one such link is

$$\Pi(k_i) = \frac{k_i}{\sum_{j=1}^{N-1} k_j}$$

Write the formula for $N(t)$ and $L(t)$: at $t=0$ the network has m_0 nodes and $L(0)$ links

Summary

Things to remember

- Preferential attachment
- How to create a BA network step by step

Practice on your own

- Describe step by step in pseudocode how to create a Barabási-Albert graph with N nodes having m_0 starting nodes and m outlinks per node.
- For your pseudocode to be valid, if at any point there is a randomized step, you must indicate **what is the probability of each possible outcome**