Closeness

Social Networks Analysis and Graph Algorithms

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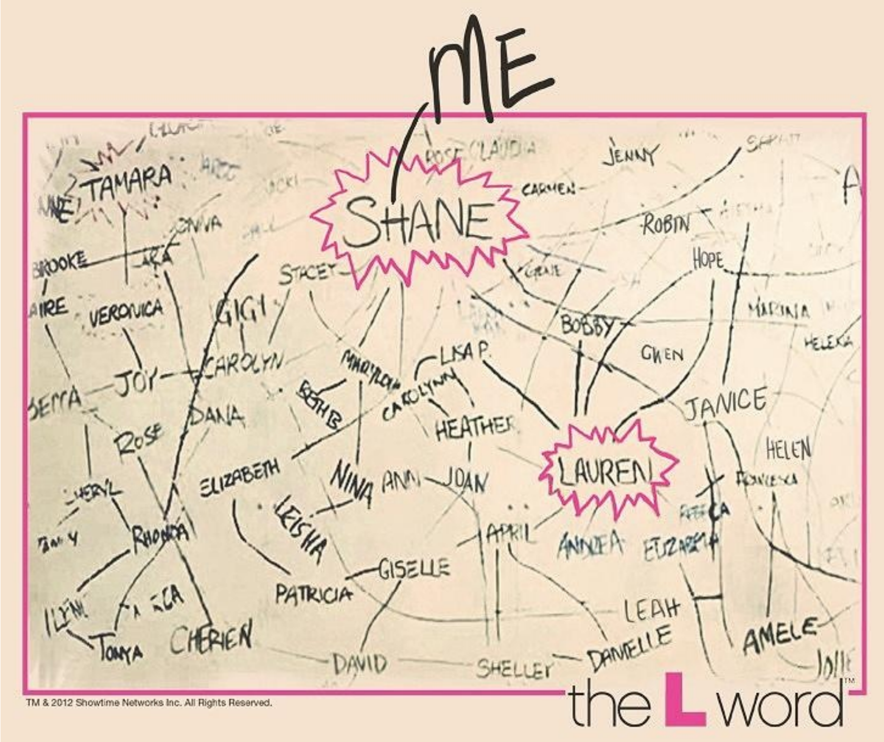
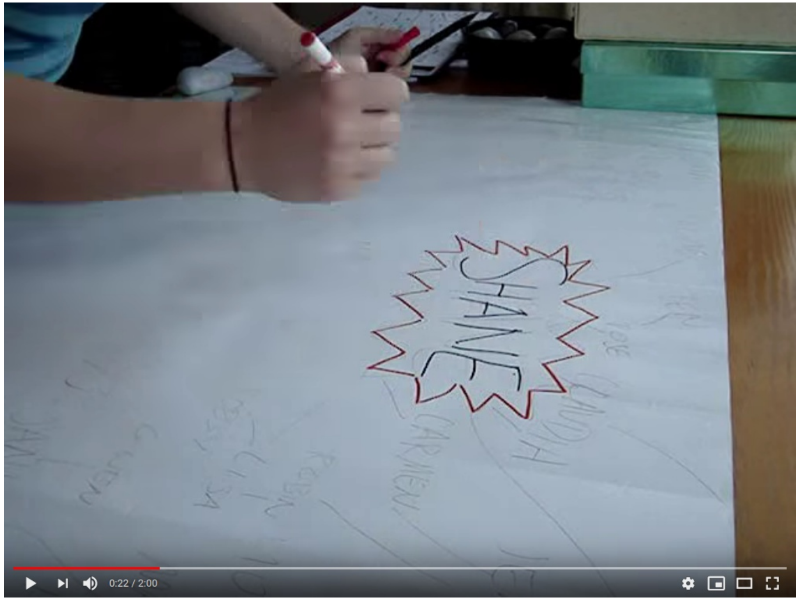
Sources

- D. Easley and J. Kleinberg (2010). Networks, Crowds, and Markets – Section 3.6B
- P. Boldi and S. Vigna (2014). Axioms for Centrality in *Internet Mathematics*.
- Esposito and Pesce (2015): Survey of Centrality.
- F. Menczer, S. Fortunato, C. A. Davis (2020). A First Course in Network Science – Chapter 02

A *central* question in networks is determining who is more … *central*





https://youtu.be/wQ3TX65MnjM?t=22

“We are all connected through love, loneliness, or one tiny lamentable lapse of judgment”

Types of centrality measure

- **Non-spectral**
  - Degree
  - Closeness and harmonic closeness
  - Betweenness
- Spectral
  - HITS
  - PageRank

Is u a well-connected person?

- Degree: u has many connections
- **Closeness:** u is close to many people
  - Average distance from u is small
- **Betweenness:** many connections pass through u
  - Large number of shortest paths pass through u
- PageRank: *u is connected to the well-connected*

Closeness

Closeness

- Distance between two nodes is $d(u,v)$

- **Closeness** is the reciprocal of the sum of distances

$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u,v)}$$

- Some graphs are not connected, in that case $d(u,v)$ can be ∞; assuming $1/\infty = 0$ one can define the **harmonic closeness**:

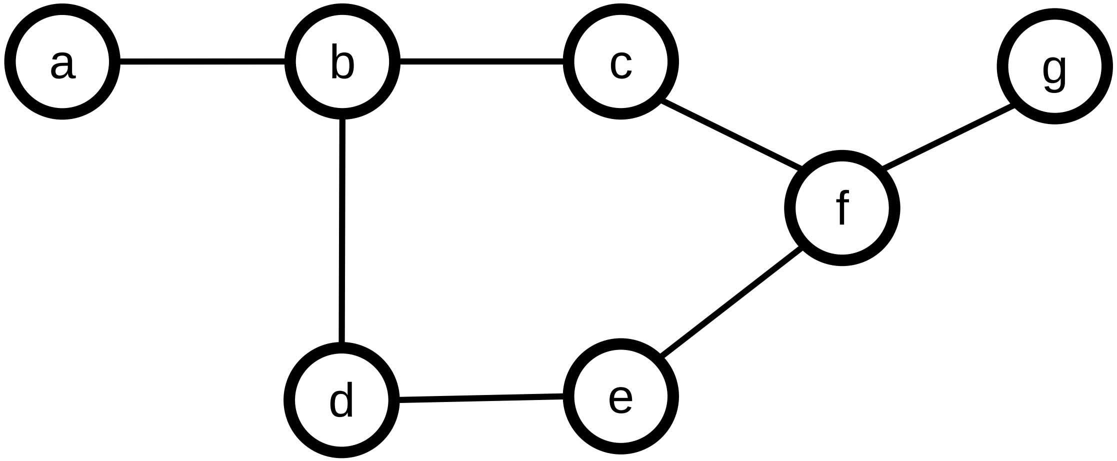
$$\text{hcloseness}(u) = \sum_{v \neq u} \frac{1}{d(u,v)}$$

Exercise

$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u,v)}$$

Compute closeness and harmonic closeness for all the nodes; d(u,v) = 1 if v is a neighbor of u

$$\text{hcloseness}(u) = \sum_{v \in V, v \neq u} \frac{1}{d(u,v)}$$



Spreadsheet links: https://upfbarcelona.padlet.org/chato/shyq9m6f2g2dh1bw

Summary

Things to remember

- Closeness and harmonic closeness definitions
- Try to compute them on your own on a graph

Constructive problems

- Practice drawing examples of graphs in which a chosen node has high degree but low closeness, or viceversa
- Can you find a graph in which there is a node that has the maximum degree and the minimum closeness? If not, why?