Connectivity in graphs

Social Networks Analysis and Graph Algorithms

Prof. Carlos Castillo — https://chato.cl/teach

Universitat Pompeu Fabra Barcelona

Contents

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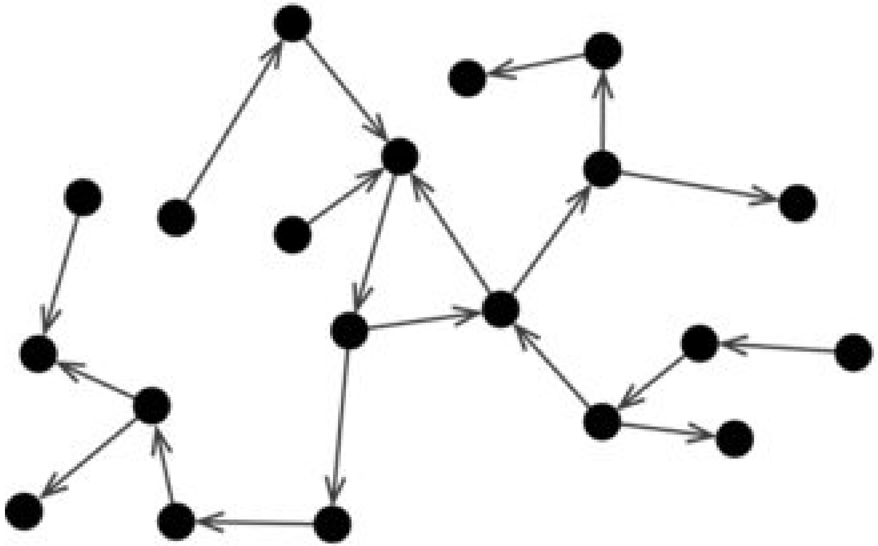
Sources

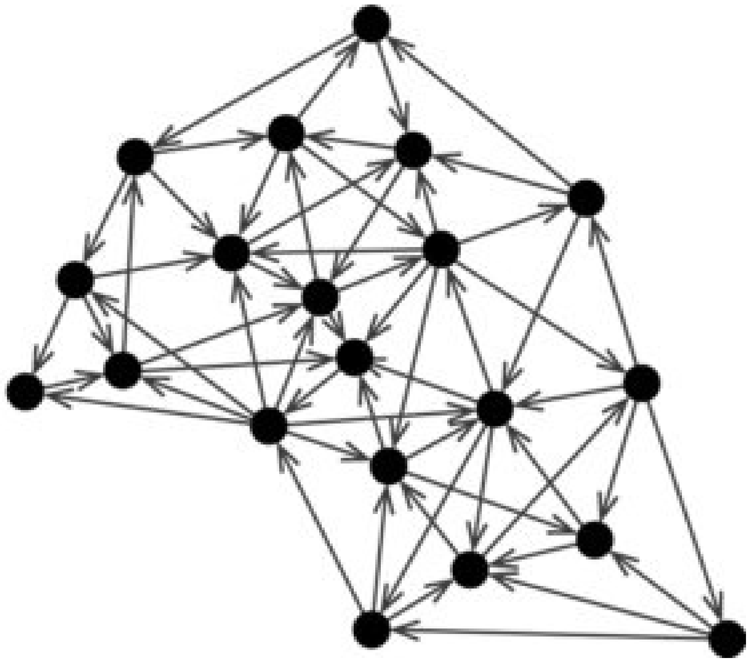
- A. L. Barabási (2016). Network Science - Chapter 01
- F. Menczer, S. Fortunato, C. A. Davis (2020). A First Course in Network Science – Chapter 02
- URLs cited in the footer of specific slides

Sparsity

Sparse network



Dense network



[Source]

Real networks are sparse

- Theoretically $$L_{\max} = \binom{N}{2} = \frac{N(N-1)}{2}$$

- Most real networks are sparse, i.e., $L \ll L_{\max}$

L is the number of links in the network, N is the number of nodes on it

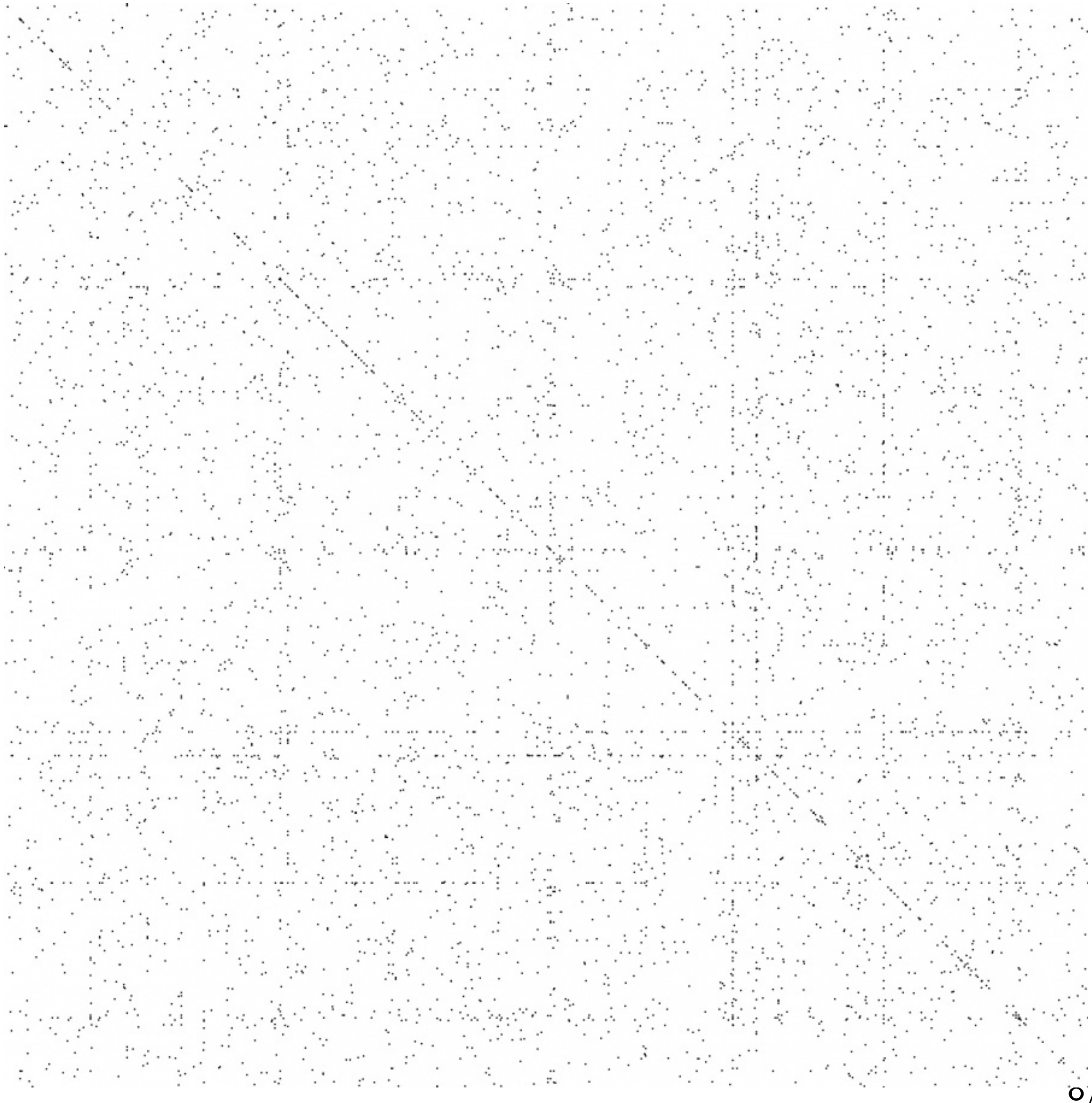
How sparse are some networks?

Network	|V|	|E|	Max |E|
Zachary's Karate Club	34	78	561
Game of Thrones	84	216	3496
US companies ownership	1351	6721	911K
Marvel comics	6K	167K	17M

https://github.com/chatox/networks-science-course/tree/master/practicum/data

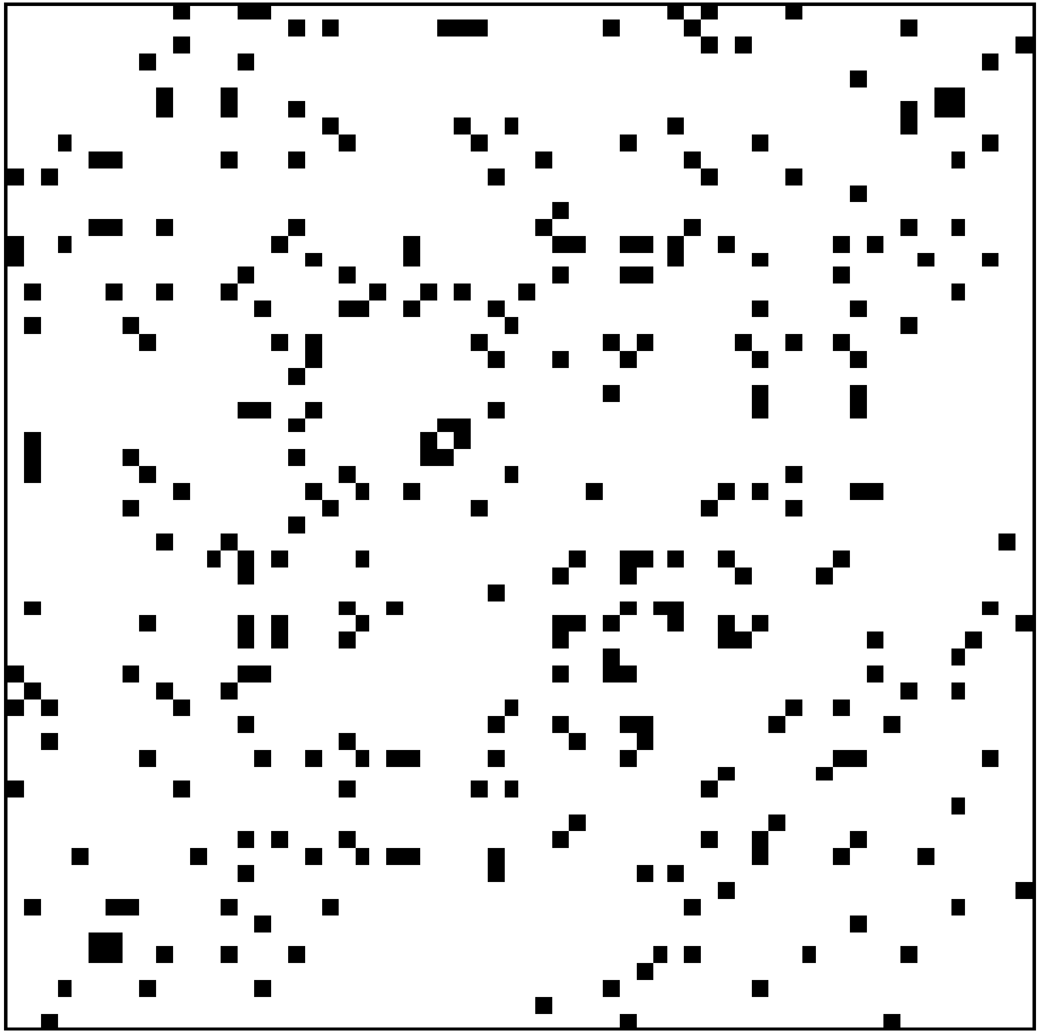
Example: protein interaction network

(N=2K, L=3K)



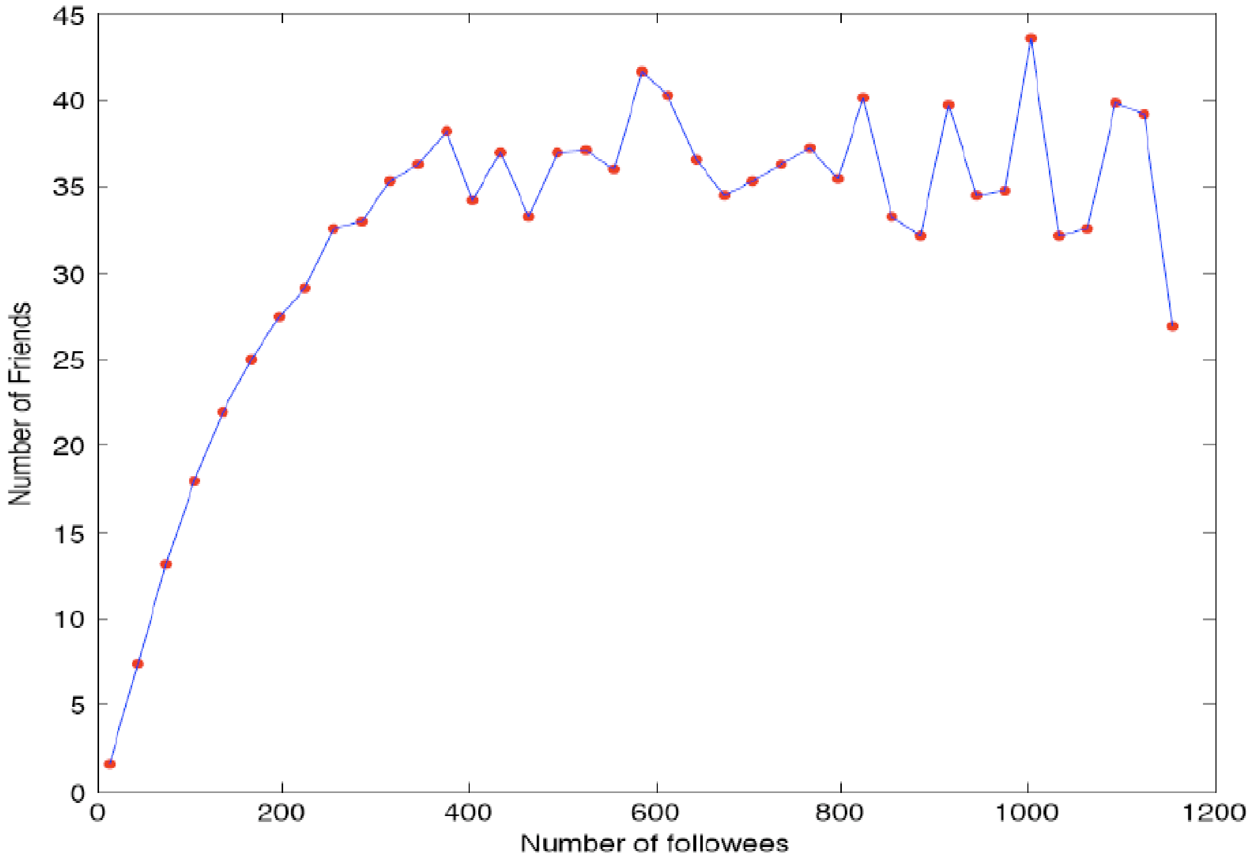
Example: dolphins

(N=62, L=318)



https://www.cise.ufl.edu/research/sparse/matrices/Newman/dolphins.html

Example: people you follow on Twitter (followees) vs people you have sent at least two messages to ("friends")

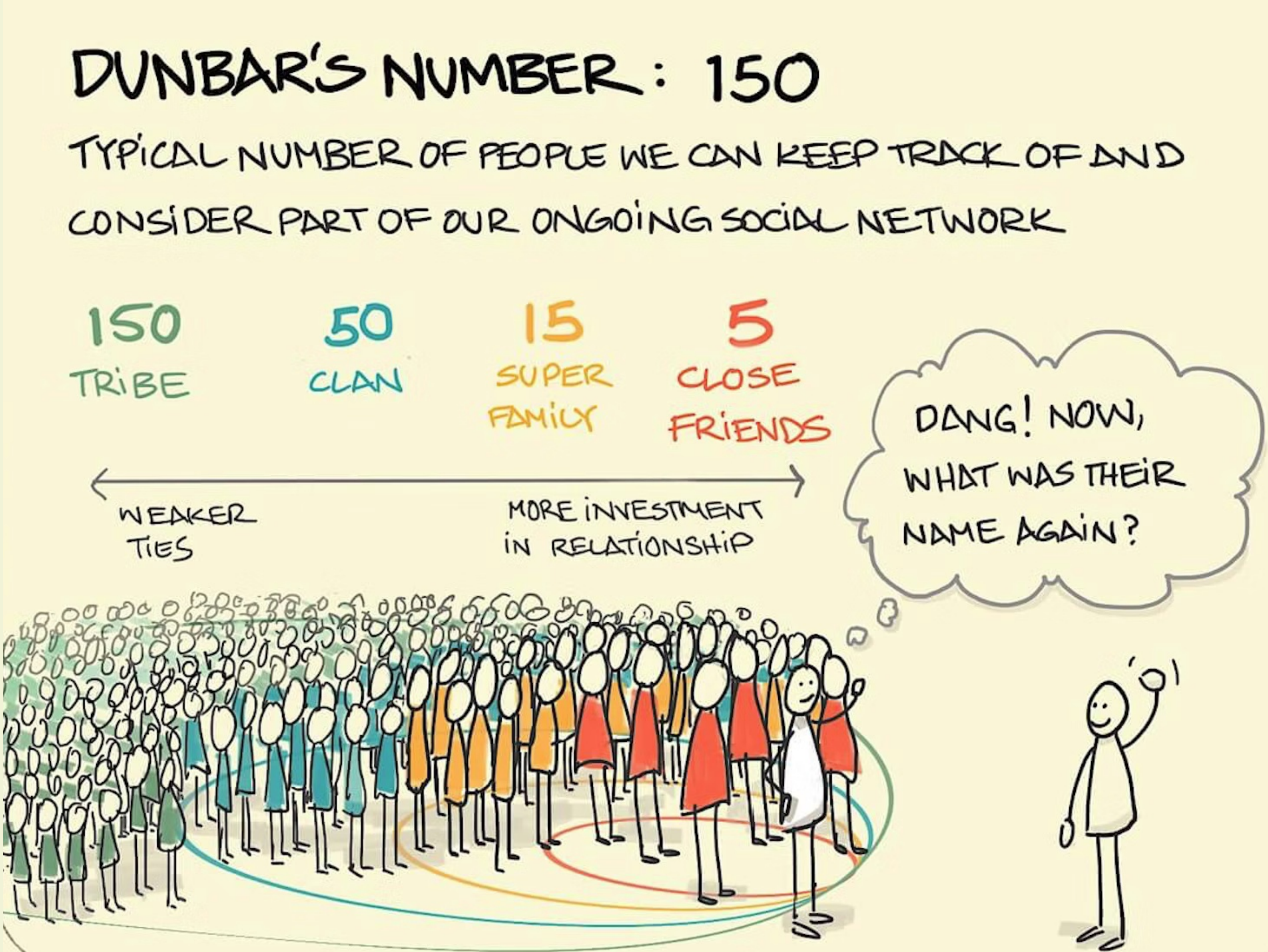


Huberman, B., Romero, D. M., & Wu, F. (2008). Social networks that matter: Twitter under the microscope. First Monday, 14(1).

Why are networks sparse?

- Different mechanisms, think about it from the node perspective:
  - How many items **could** the node be connected to
  - Would it be **realistic** to connect to a large fraction of them?
- In social networks, Dunbar's number ($\simeq$150)

https://en.wikipedia.org/wiki/Dunbar's_number

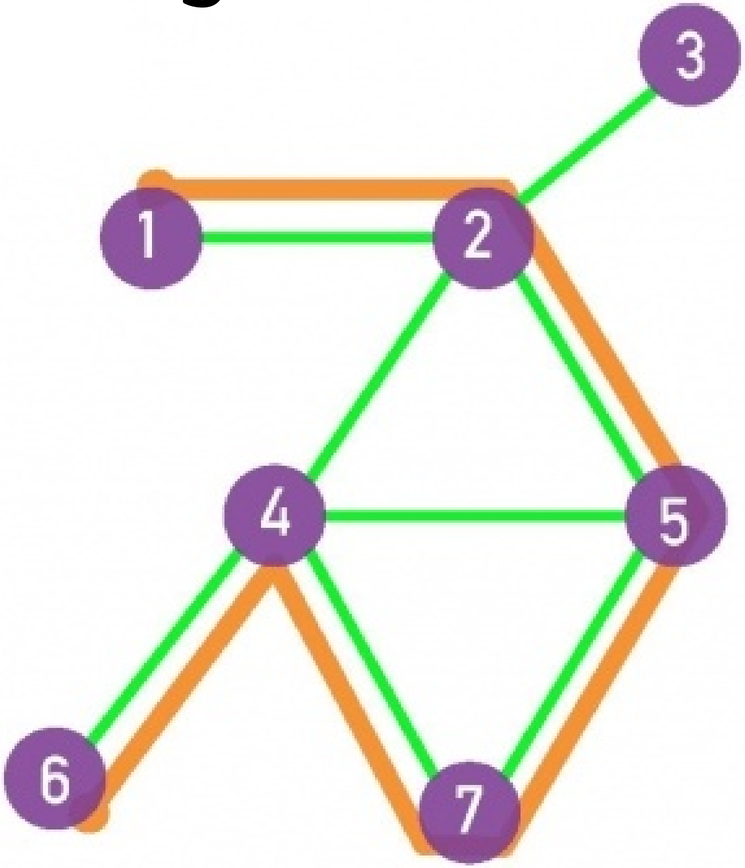


[Sketchplanations]

Paths and distances

Paths: sequences of edges

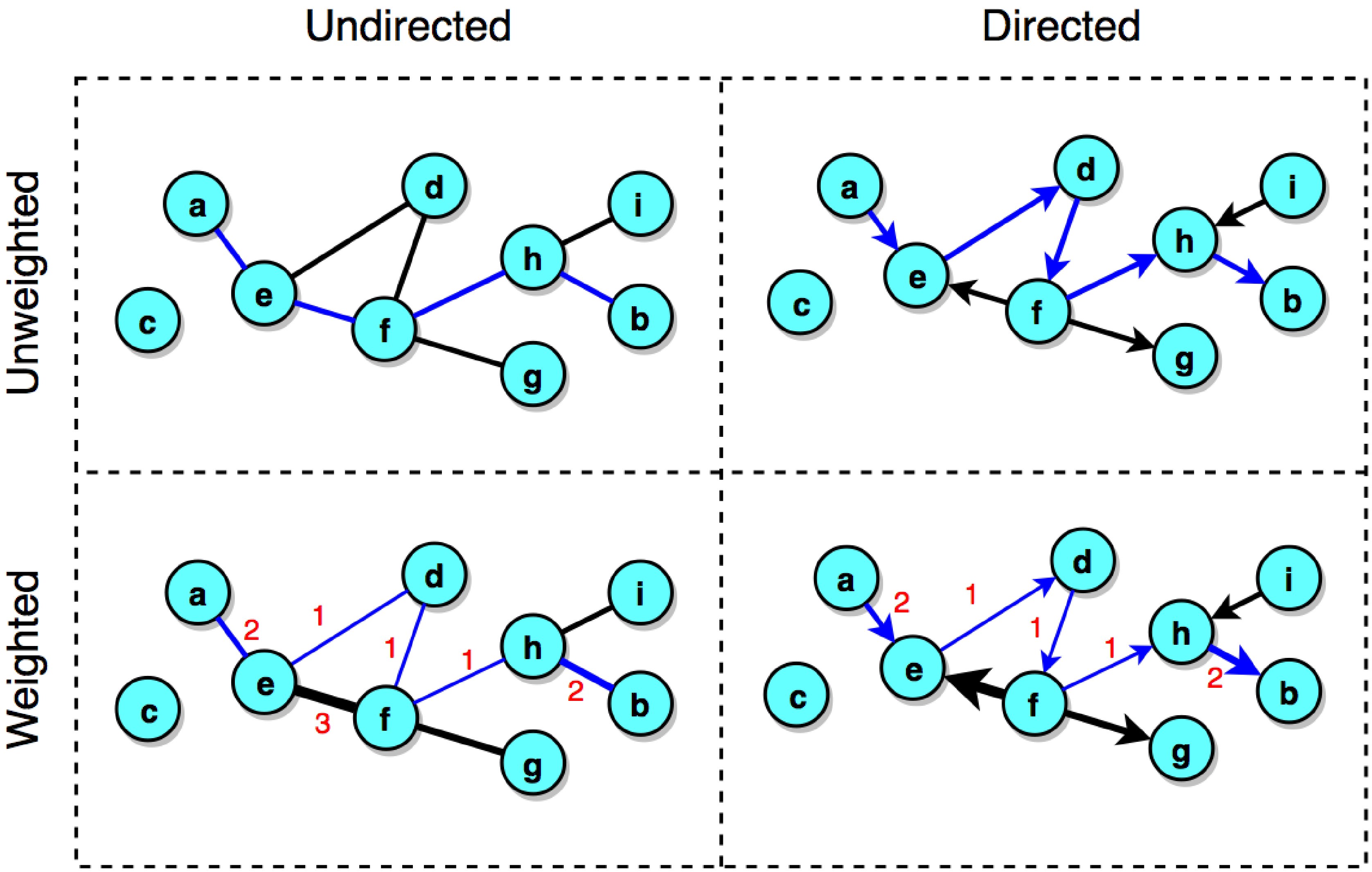
- The destination of each edge is the origin of the next edge
  - In directed graphs, paths follow the direction of the edges
- The length of the path is the number of edges on it
  - Example: path in orange has length 5



Distance

- If two nodes i, j are in the same connected component:
  - the **distance** between i and j, denoted by d_{ij} is the **length of the shortest path** between them
- If they are not in the same connected component, the distance is by definition infinite (∞)

Blue = shortest path between nodes *a* and *b*



Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Diameter

- The **diameter** of a network is the maximum distance between two nodes on it, d_{max}
- The **effective diameter** (or effective-90% diameter) is a number d such that 90% of the pairs of nodes (i,j) are at a distance smaller than d
- The **average distance** is $<d>$, and is measured only for nodes that are in the same connected component

Connected components

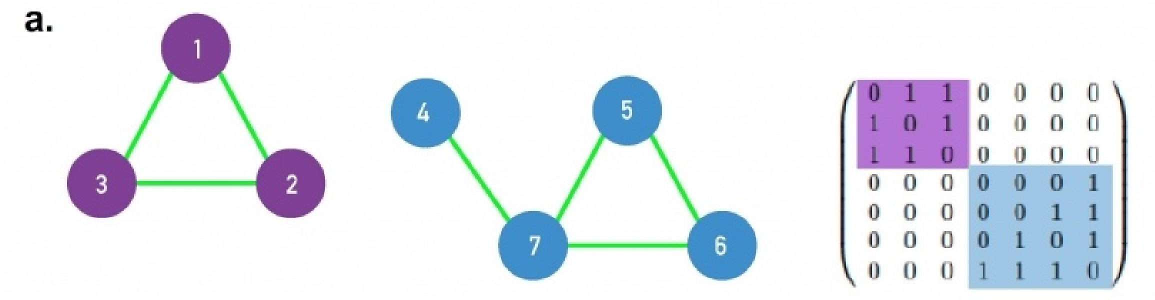
Connectedness

- If a path exists between two nodes i, j: those nodes are part of the same **connected component**
- A **connected graph** has only one connected component
- A **singleton** is a connected component with only one node

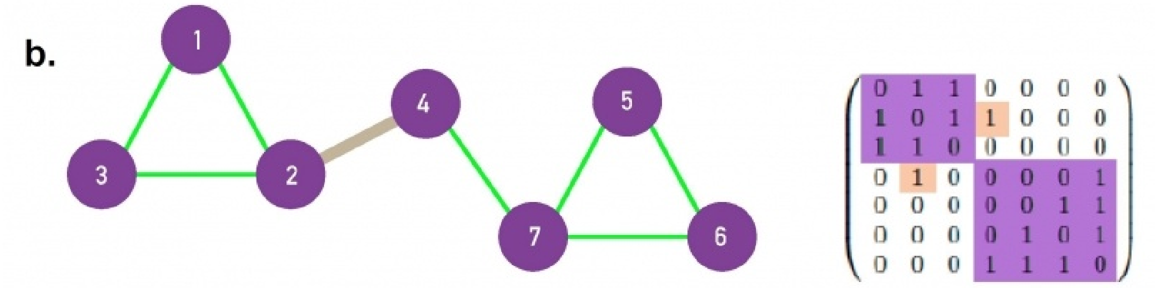
Connected graphs

A disconnected graph has an adjacency matrix that can be arranged in block diagonal form

a. disconnected



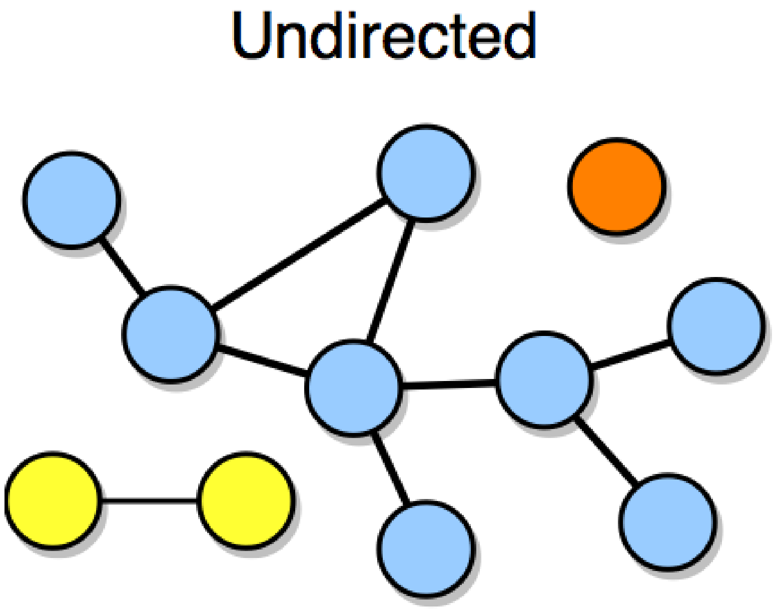
b. connected



Connectedness in directed graphs

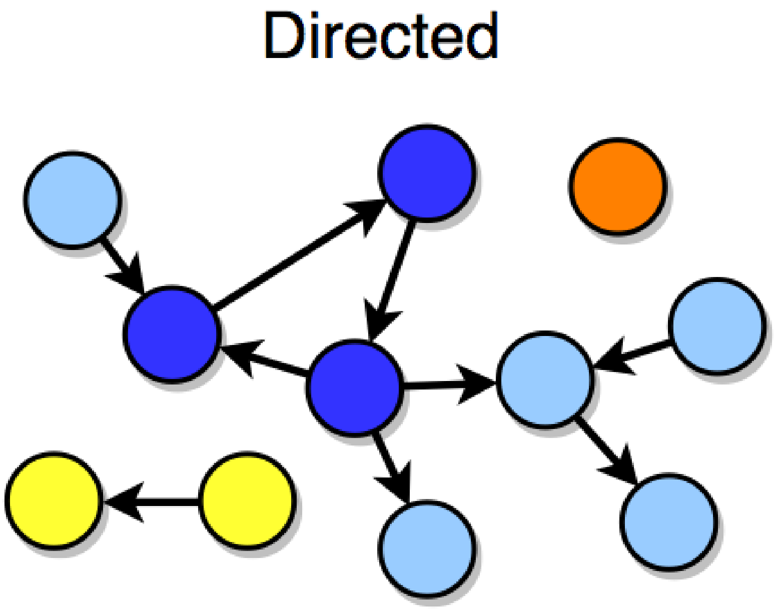
- A directed graph is **strongly connected** if it has only one connected component
- A directed graph is **weakly connected** if, when seen as undirected, has only one connected component

Connectedness example (undirected)



- Is not connected
- Has 3 connected components
- One of the connected components is a singleton

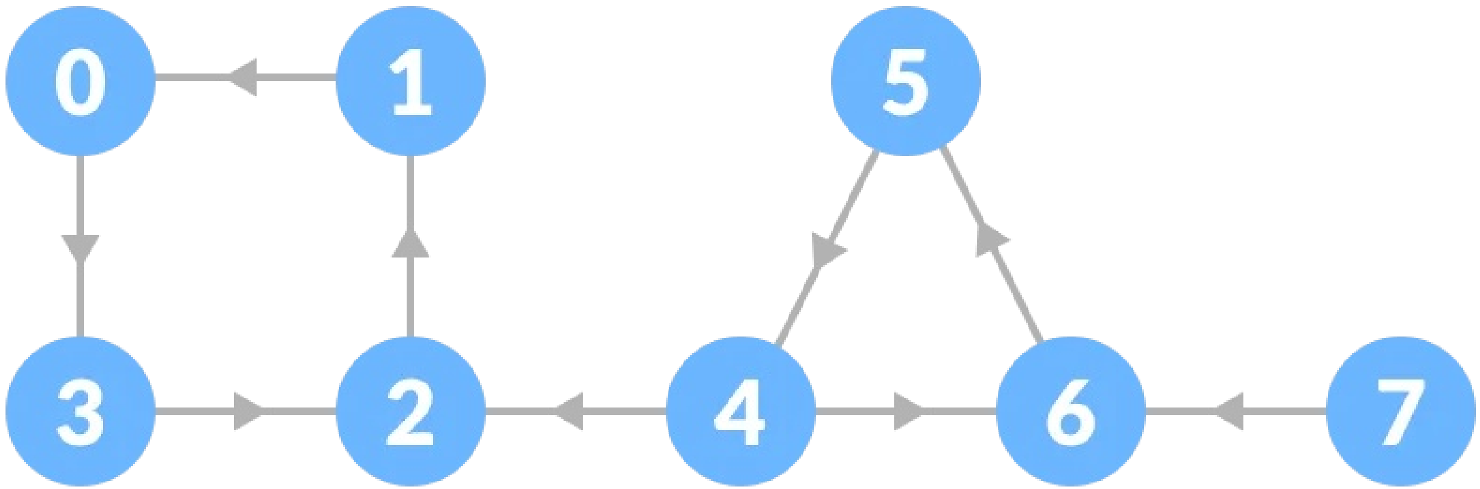
Connectedness example (directed)



- Is not strongly connected
- Is not weakly connected
- Has 3 connected components

Quick exercise

Find the strongly connected components



https://www.programiz.com/dsa/strongly-connected-components

Summary

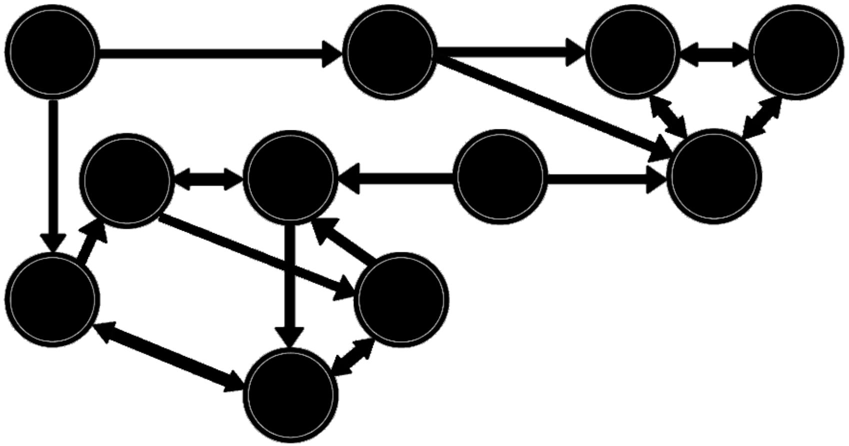
Things to remember

- Sparse vs dense graph
- Distance, diameter, effective diameter
  - In directed and undirected graphs
- Connected components
  - In directed and undirected graphs

Practice on your own

- Measure the sparsity of this graph $L/L_{\max}$

  (ignore direction of links)



Practice on your own (cont.)

- Compute the distance between two nodes
- Compute the diameter of a graph
- Identify connected components