NAME	Uxxxxxx	GRADE

Introduction to Networks Science (2025-2026)

FINAL EXAM

WRITE YOUR ANSWERS BRIEFLY and CLEARLY IN THE BLANK SPACES. Underline key words in your answers. If you include intermediate calculations, circle the final result.

Problem 1 *0.5 point*

Consider the Erdős–Rényi (ER) network with $N = 200000$ nodes. For which approximate value of p, the connection probability between pairs of nodes, would you expect to observe a phase transition between mostly disconnected nodes and the emergence of a giant connected component in the network? Motivate your answer.

Problem 2 *1 point*

Consider a Scale-Free network with degree distribution $P(k) \sim k^{-\gamma}$ (with $\gamma = 2.5$). We observe a "Hub": a node with the largest degree k_{max}. We double the size of the network (from N to $2N$). Calculate the variation in the expected value of the degree of the Hub, $k_{max}(2N)/k_{max}(N)$.

Problem 3 *2 point*

Consider a fully connected network of size N, with N even. Consider a graph bisection problem: we want to divide the network into two groups of nodes with the same size ($N/2$), such that we minimize the cut size (i.e., the number of links connecting the two groups).

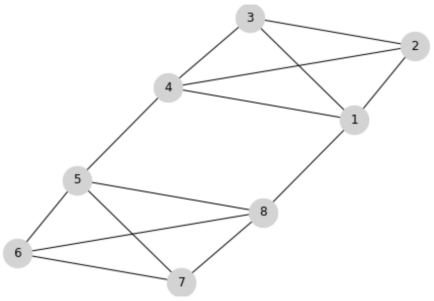
1. What is the cut size of the resulting bipartition? (1pt)

2. How many optimal solutions (unique configurations) are possible? (1pt)

Problem 4 *1.5 point*

Consider a threshold model on this small graph. The threshold for activation is $\theta = 0.5$ for all nodes.

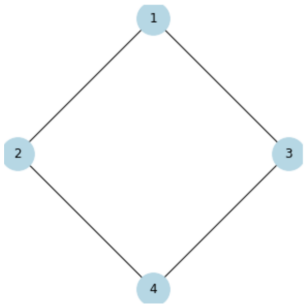
1. If we initially activate Nodes 1, 2, how large is the final cascade? List the active nodes. (0.5 pts)



2. Which is a minimal set of seed nodes to activate, in order to activate the whole network? Explain why. (1 pt)

Problem 5 *4 points*

Consider this small graph of 4 nodes.



1. Write the Laplacian matrix L for this graph (0.5 pts).

2. Calculate the eigenvector v_a corresponding to the smallest eigenvalue λ_a. (0.5 pts)

3. Consider the eigenvector $v_b = (0, 1, x_3, 0)^T$. Based on the properties of the eigenvectors, calculate the value of x_3. Calculate the corresponding eigenvalue λ_b. (1pt)

4. Consider the eigenvector $v_c = (y_1, 1, 1, y_4)^T$. Based on the orthogonality of eigenvectors and the symmetry of the graph, calculate the values of y_1 and y_4. Calculate the corresponding eigenvalue λ_c. (1pt)

5. Which of the three eigenvectors (v_a, v_b, v_c) would you use as one-dimensional coordinates to partition the graph? Explain your reasoning. (1pt)

Problem 6 *1 point*

Consider an epidemic spreading on a homogeneous network with average degree $\langle k \rangle$, with basic reproduction number $R_0 = 2.5$.

1. Let us assume that we develop medicines that can double the recovery rate μ (i.e., $\mu_{new} = 2\mu$). Calculate the new R_0. Does the epidemic stop? (0.5pt)

2. Ignoring the medicine from step 1 (i.e., starting from the original $R_0 = 2.5$), calculate by how much the average degree $\langle k \rangle$ should be reduced to stop the epidemic spreading. (0.5 pts)