NAME	Uxxxxxx	GRADE

Introduction to Networks Science (2023-2024)

FINAL EXAM

WRITE YOUR ANSWERS BRIEFLY and CLEARLY IN THE BLANK SPACES. Please: underline keywords in your answers, include intermediate calculations, and circle the final result. Please use only capital letters.

Problem 1 — *0.5 point*

Consider the Erdős–Rényi network model. You have $N = 50$ nodes and want the average node degree to be 5. What approximate value for p, the probability of connecting two nodes, would you use and why? Show your calculations.

Problem 2 — *1 point*

Consider the Erdős–Rényi (ER) and the Barabasi-Albert network models. Fill in the following table, indicating if each model is approximately able (GOOD), or not able (BAD), to capture the given property of most real networks.

	Degree distribution	*Clustering Coefficient*	*Average Path Length*
ER model			
BA model			

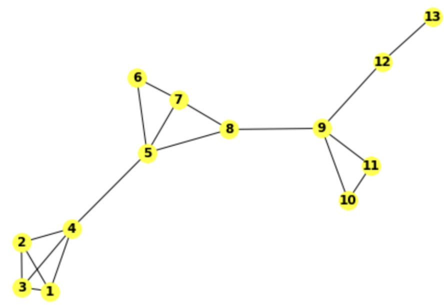
Problem 3 — *1 point*

You want to attack Facebook. You produced a computer virus that deletes the accounts who open it. Your goal is to reach accounts with many friends and delete as many as possible.

Which attack is more likely to be effective: 1) sending the virus to randomly chosen accounts, or 2) sending the virus to a friend of randomly chosen accounts? Explain why. State the assumptions you made in your reasoning.

Problem 4 *1 point*

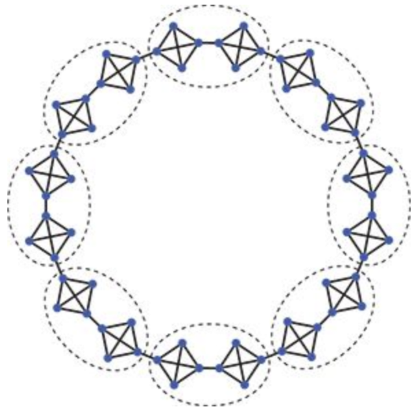
Perform a k-core decomposition of the graph shown on the right, indicating clearly which nodes belong to the 1-core, 2-core, 3-core, ...



Node	1	2	3	4	5	6	7	8	9	10	11	12	13
Core													

Problem 5 *1 points*

Consider this graph and two partitions: Partition A in which each clique of four nodes is a community (16 communities), and Partition B where a pair of cliques is a community (8 communities, circled for clarity).



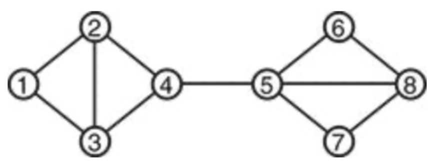
Recall: the formula for modularity is $Q = \frac{1}{L} \sum_C \left(L_C - \frac{k_C^2}{4L} \right)$ where C are the communities, L is the total number of links, L_C is the number of links internal to community C, and k_C is the summation of the degree of the nodes in C.

- Calculate the modularity of partition A and partition B. *Hint*: Use the symmetry of the graph. (0.5 pts)

- Which partition would the Louvain algorithm choose and why? (0.5 pts)

Problem 6 *1.5 point*

Consider the fractional threshold model on this small graph. The threshold is 1/2 for all nodes. A cascade is defined by all activated nodes at the end of the dynamics. Answer the following questions, justifying your replies.



- How large is the cascade if we initially activate node 1? (0.25 pts)

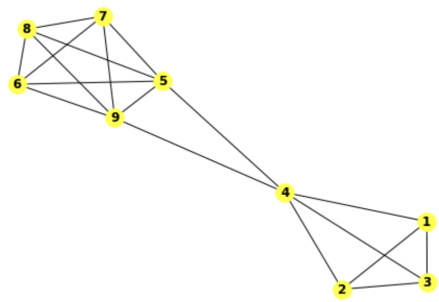
- Which node should we initially activate to obtain the largest possible cascade? (0.5 pts)

- Is the solution unique? (0.25 pts)

- What is the minimum number of initial nodes needed to activate the whole network? (0.5 pts)

Problem 7 *2 points*

Consider this small graph.



- Write its Lapcian. (0.5 pts)

- Write the eigenvector corresponding to the smallest eigenvalue (0.5 pts)

The eigenvectors corresponding to the second and third smallest eigenvalues of the Laplacian are $v_2 = (-0.42, x_2, -0.42, -0.18, 0.23, x_6, 0.33, 0.33, 0.23)$, $v_3 = (-0.12, y_2, -0.12, 0.68, -0.44, y_6, 0.18, 0.18, -0.44)$, where x_2, x_6, y_2, y_6 are unknown values.

- Use v_2 and v_3 to embed the graph in a plane, except for nodes 2 and 6. (0.5 pts)

- Based on the symmetry of the graph, can you tell the values x_2, x_6, y_2, y_6? (0.5 pts)

Problem 8 *2 points*

Consider a virus, with infection rate $\beta = 0.1$ and recovery rate $\mu = 0.3$, spreading by the Susceptible-Infected-Susceptible dynamics on two different contact networks, A and B. Network A is a homogeneous network, where all nodes have the same degree $\langle k \rangle = 4$. Network B is a heterogeneous network, whose degree distribution follows a power law, $P(k) \sim k^{-\gamma}$, with $\gamma = 2.5$. Both networks can be assumed infinitely large.

- Would you expect the virus to be able to spread in network A? Why? (0.5 pts)

- Would you expect the virus to be able to spread in network B? Why? (0.5 pts)

Suppose you immunize a fraction $g = 1/2$ of the population, choosing at random the individuals to immunize.

- Now, would you expect the virus to be able to spread in network A? Why? (0.5 pts)

- Now, would you expect the virus to be able to spread in network B? Why? (0.5 pts)