NAME	Uxxxxxx	GRADE

Introduction to Networks Science (2023-2024)

MID-TERM EXAM

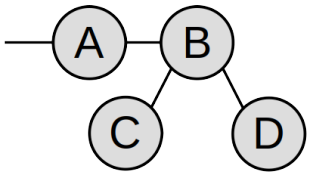
WRITE YOUR ANSWERS BRIEFLY and CLEARLY IN THE BLANK SPACES. Please underline key words in your answers. Please if you include intermediate calculations, circle the final result. If for some reason (e.g., if after you have written the solution you realize that there is some mistake), you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet.

Problem 1

1 point

A *dangling* edge is a special kind of edge that has one end in a node, while the other end is not attached to anything. In the example in the figure, node A has a dangling edge.

According to the definition of *subgraph* we saw in class, can a subgraph have dangling edges?



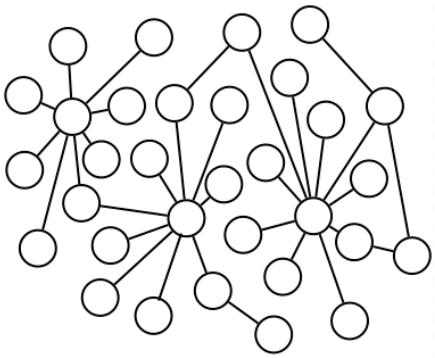
Your answer and brief justification:

Problem 2

1 point

Draw the degree distribution of the graph on the right.

Bar plot with the degree distribution:



Problem 3

1 point

Let us define the *sparsity* of a graph $G = (V, E)$ as a number from 0 to 1 indicating the ratio between the number of links in the graph, $|E|$, and the maximum number of links that a graph with the same number of nodes $N = |V|$ could have.

What is the sparsity of the graph in the previous problem? Define any variable of formula you use, i.e., do not use variables of formulas without explaining what they mean:

Problem 4 *1 point*

You are given an undirected *lattice graph* containing N^2 nodes. We would like to know what is the average degree in this graph, $\langle k \rangle$, as a function exclusively of N. Briefly justify your answers.

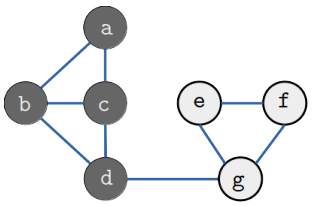
What is the average degree in this graph, as a function of N?

What is this in the limit as N tends to infinity?

Problem 5 *2 points*

Consider the graph on the right, composed of 4 black nodes (we will name this group B) and 3 white nodes (we will name this group W).

Compute the homophily of both groups (include your calculations), and indicate if each group is homophilic, heterophilic, or neutral. Define any variable that you use, do not use variables without defining them.



Answer:

Tip: to help you remember the formula, the homophily of a group is the ratio between the number of inter-group links, and the expected number of inter-group links of that group. When we have two groups A, B, the expected number of inter-group links of group A is the product of the total number of links of group A and the fraction of nodes in the graph that belong to group B.

Problem 6 *1 point*

Consider a *bi-partite* graph $G = (V, E)$ in which the nodes can be partitioned into V_1 and V_2, with $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$, and $E \subseteq V_1 \times V_2$. Let $N_1 = |V_1|$ and $N_2 = |V_2|$, so $N = N_1 + N_2$.

If the graph is a *bi-partite clique*, what is the clustering coefficient of the nodes in V_1?

Your answer and justification:

Problem 7 *1 point*

Consider the same *bi-partite clique* of the previous problem and the same notation introduced there. What are the hub scores and authority scores of nodes in V_1? What are the hub scores and authority scores of nodes in V_2? We refer to the normalized values, in both cases.

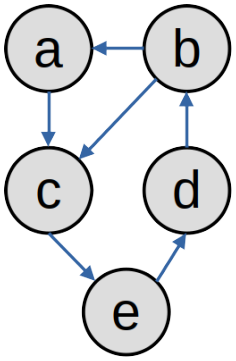
Normalized hub scores and authority scores of nodes in V_1?

Normalized hub scores and authority scores of nodes in V_2?

Justification for your answers:

Problem 8 *2 points*

Perform 4 iterations of *Simplified PageRank* for the graph on the right. Please express your calculations in *decimal notation* (not as fractions), with 3 digits after the decimal period.



Answer:

Node	Init	Iter 1	Iter 2	Iter 3	Iter 4
a					
b					
c					
d					
e					