NAME	NIS (uXXXX) or NIA	GRADE

Introduction to Networks Science (2022-2023)

FINAL EXAM

WRITE YOUR ANSWERS <u>BRIEFLY</u> and <u>CLEARLY</u> IN THE BLANK SPACES. Please underline key words in your answers. Please if you include intermediate calculations, circle the final result. If for some reason (e.g., if after you have written the solution you realize that there is some mistake), you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet.

Problem 1 — *1 point*

Consider an ER graph with $N = 100,000$ nodes. Answer these questions, indicating the formula you are using.

What is the minimum integer such that if the average degree $\langle k \rangle$ is that integer, the graph is in ***connected regime****?*

What is the clustering coefficient we would expect nodes would have in this case?

Problem 2 — *1 point*

Describe the differences in the degree distribution and average distances between a random network (ER), and a scale-free network with γ between 2 and 3. Complete the following, using formulas when appropriate:

Differently from an ER network, in a scale-free network with $\gamma \in]2, 3[$, the degree distribution ...

Differently from an ER network, in a scale-free network with $\gamma \in]2, 3[$, the distances ...

Problem 3 — *1 point*

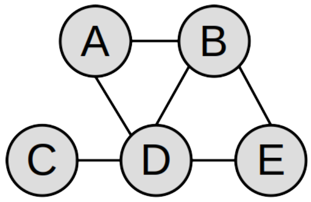
Find a graph that you can describe concisely (e.g., by referring to one of the graphs that we have named through the course), in which most nodes have degree 4, but all nodes are in the 2-core. Explain why. **Do not draw the graph.**

An example graph with the above property is:

Tip: If your description and explanation do not fit above, or you need more space, think of a different graph. To get inspiration, think of a graph in which most nodes have degree 2, but all nodes are in the 1-core.

Problem 4 *2 points*

Suppose we are running the preferential attachment process with $m = 2$ and so far we have the graph on the right. We execute one step of the process, adding a node F.



The expected degree of each node after adding node F is (use 4 decimals):

- $\langle k_A \rangle =$

- $\langle k_B \rangle =$

- $\langle k_C \rangle =$

- $\langle k_D \rangle =$

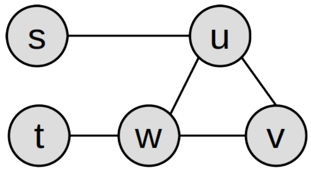
- $\langle k_E \rangle =$

- $\langle k_F \rangle =$

Use this space for calculations:

Problem 5 *2 points*

Suppose we want to partition the graph on the right into two communities, community 1 containing u and v, and community 2 containing s and t. According to the modularity criterion, to which community should node w belong?



The formula for modularity Q is $Q = \frac{1}{L}\sum_C \left(L_C - \frac{k_C^2}{4L}\right)$ where C are the communities, L is the total number of links, L_C is the number of links internal to community C, and k_C is the summation of the degree of the nodes in C.

Modularity of partitioning in which w goes to community 1:

Modularity of partitioning in which w goes to community 2:

Hence, according to this criterion, node w should be in community:

Problem 6 *1 point*

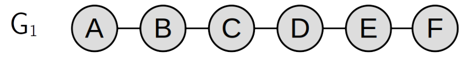
Consider a **line graph** with $N+1$ nodes $0, 1, 2, \ldots, N$. Consider a propagation starting at node 0 using the independent cascade model in which the propagation probability of all N edges is p. What is the probability that the number of infected nodes is exactly x, for $1 < x < N$? Justify your answer.

The probability that the number of infected nodes is x is:

Justification:

Problem 7 *2 points*

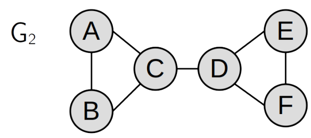
Write L^{G_1}, the Laplacian of graph G_1:



The eigenvector of the second smallest eigenvalue of the Laplacian of graph G_1 is $v_2^{G_1} = [-0.5577, -0.4082, -0.1494, 0.1494, 0.4082, 0.5577]$.

Determine $\lambda_2^{G_1}$, the second smallest eigenvalue of the Laplacian of graph G_1:

Write L^{G_2}, the Laplacian of graph G_2:



The eigenvector of the second smallest eigenvalue of the Laplacian of graph G_2 is $v_2^{G_2} = [-0.4647, -0.4647, -0.2610, 0.2610, 0.4647, 0.4647]$.

Determine $\lambda_2^{G_2}$,the second smallest eigenvalue of the Laplacian of graph G_2:

According to the criterion we saw in class regarding the second smallest eigenvalue, which graph is more naturally partitioned into two communities? Why?