| NAME | NIA | GRADE |
| :--- | :--- | :--- |

Introduction to Networks Science (2022-2023)
$\qquad$

WRITE YOUR ANSWERS BRIEFLY and CLEARLY IN THE BLANK SPACES. Please underline key words in your answers. Please if you include intermediate calculations, circle the final result. If for some reason (e.g., if after you have written the solution you realize that there is some mistake), you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet.

## Problem 1

What is an emerging property? Give an example in the context of a complex network.
Answer:

## Problem 2

You are given an undirected line graph of N nodes.
What is the average degree on this graph? Justify your answer.
Answer:

## Problem 3

You are given an adjacency matrix $A=\left(a_{i j}\right)_{N \times N}$ for a directed graph $G$ of $N$ nodes.
Write a formula for the average out-degree of graph $\left\langle k^{o u t}\right\rangle$ and for the average in-degree $\left\langle k^{i n}\right\rangle$ of graph $G$, as a (potentially nested) sum. Remember $a_{i j}$ contains a 0 or 1 depending on whether a link from node $i$ to node $j$ exists.

Answer:

Consider a 3-dimensional lattice graph, similar to the one shown in the figure, containing $n^{3}$ nodes with $n>3$. Observe that there are no nodes with degree 0,1 , or 2 . Complete the degree frequenty table below:

## Answer:

| Degree | Number of nodes |
| :--- | :--- |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |



## Problem 5

1 point
What is the diameter of the lattice of the previous question? Justify your answer.
Answer:

## Problem 6

1 point
Consider the graph on the right, composed of 5 black nodes (we will name this group $B$ ) and 4 white nodes (we will name this group $W$ ).

Compute the homophily of both groups (include your calculations), and indicate if each group is homophilic, heterophilic, or neutral.

Answer:


Compute the clustering coefficient of all white nodes of the previous graph $\{a, b, c, d\}$ in their original graph (i.e., NOT as a subgraph). Write your results as simplified fractions.
Answer:
$C_{a}=$
$C_{b}=$
$C_{c}=$
$C_{d}=$

Problem 8
1 point
Use the Brandes-Newman algorithm to compute the edge betweenness of all the edges in this graph. Mark clearly your final answer.

Answer:


Complete the table below for computing Hubs and Authorities for the graph shown on the right using the procedure seen in class. Use decimal numbers with 3 digits, or simplified fractions.

Answer:

|  | $\hat{H}(1)$ | $A(1)$ | $\widehat{A}(1)$ | $\mathrm{H}(2)$ |
| :--- | :--- | :--- | :--- | :--- |
| a |  |  |  |  |
| b |  |  |  |  |
| c |  |  |  |  |



## Problem 10

1 point
Perform two iterations of Simplified PageRank for the graph shown on the right. Indicate in the table below the initialization scores, the scores after the first iteration, and the scores after the second iteration. Use decimal numbers with 3 digits, or simplified fractions.

Answer:

|  | Initialization | Iteration 1 | Iteration 2 |
| :--- | :--- | :--- | :--- |
| A |  |  |  |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |
| E |  |  |  |



