NAME	NIA	GRADE

Introduction to Network Science (2021-2022)

MID-TERM (TT02-TT12, TT14)

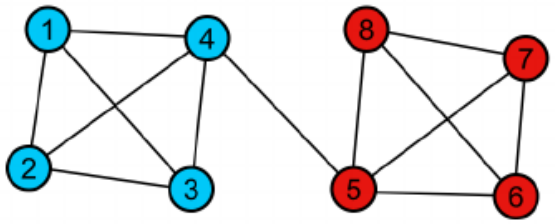
WRITE YOUR ANSWERS CLEARLY IN THE BLANK SPACES. You can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet. Also, you may use other sheets to perform your calculations. Please underline or highlight the important keywords of your answers and the final answers when you have performed a calculation.

Problem 1 — *0.6 points*

Suppose a graph is such that all nodes have even degree. Can the number of links in this graph be odd? If not, indicate why not. If yes, give an example.
Answer:

Problem 2 — *1 point*

For the graph given on the right.
(0.5 points) Draw its degree distribution as a bar plot



(0.5 points) Draw its adjacency matrix

Problem 3 — *0.4 points*

Given an adjacency matrix, how can you tell if a graph is ...
(0.2 points) ... a complete graph?

(0.2 points) ... a directed graph?

Problem 4 *1 point*

Let us define a metric of sparsity for a graph $G = (V, E)$ as $S(G) = 1 - \frac{L}{L_{\max}}$, where $L_{\max}$ is the maximum number of links that G could have.

(0.5 points) What is the formula for $L_{\max}$ given $G = (V, E)$? Briefly explain your formula.

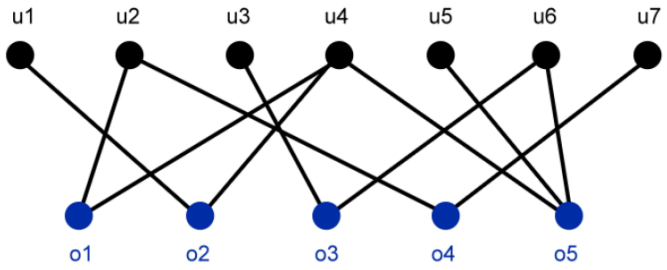
(0.5 points) Given an ER graph $G = (V, E)$ generated using parameter p, what is $S(G)$? Briefly explain your formula.

Tip: Define all the variables you use!

Problem 5 *1 point*

Draw the left-projection ("u" side) and the right-projection ("o" side) of this graph.

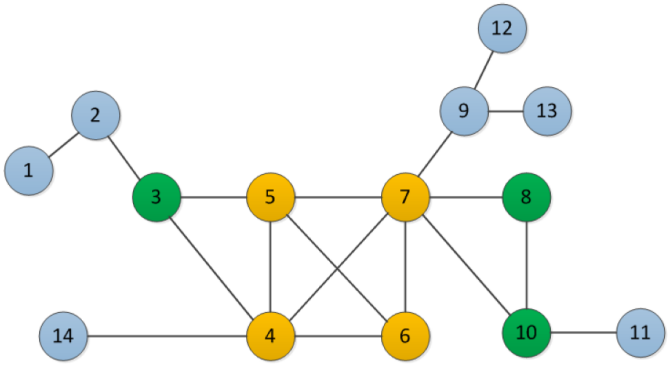
Left projection:



Right projection:

Problem 6 *1 point*

Compute the clustering coefficient of nodes 4, 5, 6, and 7 in this graph. Indicate the numerator and the denominator, and provide your answer either as a simplified fraction or in decimal notation.



Clustering coefficient of node 4:

Clustering coefficient of node 5:

Clustering coefficient of node 6:

Clustering coefficient of node 7:

Problem 7 *2 points*

Consider a graph $G = (V, E)$ having $|V| = 1,000$ nodes and generated using the ER model with linking probability $p = 0.01$.
(1 point) What is the probability that a node in G will have 0 links? Include the relevant formula(s) and briefly explain.

(1 point) What is the expected distance $\langle d \rangle$ in G? Include the relevant formula(s).

Problem 8 *1 point*

Consider a scale-free network of $N = 500,000$ nodes and $\gamma = 2.2$. The value of Riemann's Zeta of 2.2 is $\zeta(2.2) = 1.4905$.

What is the maximum degree x for which you would still expect to find at least one node with degree x?
Your answer, including the formula(s) you use:

Problem 9 *0.5 point*

Why do we say that scale-free networks with large values of γ are indistinguishable from random (ER) networks?
Your answer:

Problem 10 *0.5 point*

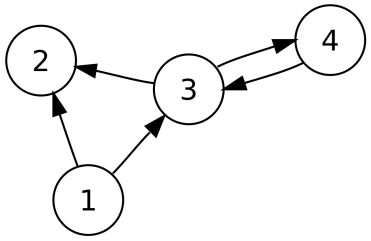
Describe the **copy model** in pseudocode.
Input parameters:

Initialization:

For every new node v:

Problem 11 *1 point*

Consider the directed graph of the figure. Transform the graph into a bipartite graph, and compute the first iterations of hubs and authorities.



Bipartite graph	$\hat{H}(1)$	$A(1)$	$\hat{A}(1)$	$H(2)$	$\hat{H}(2)$	