## Resit exam questions (2021-07-15)

## Exam protocol

- Choose language es/ca/en
- We are recording now, the recording will stay in the platform with access only to me, me unless the university authorities request it for some reason
- Please place your mobile in airplane mode (unless you're using it for communicating with me)
- Please briefly show me the room where you are giving your exam
- Please briefly share with me ("present") your entire computer screen
- We will start with a topic you think you've studied more, then we will go back to slide \#3 and roll the dice to determine each question; if we land on a question you've already answered or a non-question slide, I ask you the next one; if we get to the end we restart
- I'll ask you questions for 20 minutes starting now - pick the initial topic please


## TT01 Complex networks

## TT02. Complex networks

What is a complex system?
What is a complex network?

## TT04. Graph theory basics

What is a digraph?

## TT04. Graph theory basics

## Draw the graph corresponding to this adjacency matrix

|  | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 2 | 3 | 0 | 0 |
| 1 | 2 | 0 | 15 | 2 | 0 |
| 2 | 3 | 15 | 0 | 0 | 13 |
| 3 | 0 | 2 | 0 | 0 | 9 |
| 4 | 0 | 0 | 13 | 9 | 0 |

## TT04. Graph theory basics

Write the adjacency matrix of this network:


## TT05. Sparsity and connectivity

What is a bi-partite graph?

## TT05. Sparsity and connectivity

Why do we say most real networks are sparse?

## TT05. Sparsity and connectivity

Draw the left- and right-projection of this bipartite graph



## TT05. Sparsity and connectivity

What is the diameter of this graph?


## TT05. Sparsity and connectivity

What is the average distance in this graph?


## TT05. Sparsity and connectivity

What is the sparsity of this graph L/Lmax ?


## TT06. Clustering coefficient

Compute the local
clustering
coefficient of each
 node in this graph

## TT06. Clustering coefficient

What is a the (global) clustering coefficient of a graph?

## TT07. Random networks

Indicate what input parameters are needed and how one creates a random (ER) graph

## TT07. Random networks

What is the maximum degree in an ER graph of $N$ nodes and edge probability p?

## TT07. Random networks

What is the expected number of links in an ER graph of $N$ nodes and edge probability $p$ ?

## TT07. Random networks

What probability distribution follows the degree in an ER network?

## TT07. Random networks

If an ER graph has average degree <k> and $N$ nodes, what is its linking probability $p$ ?

## TT08. Properties of rand. networks

 Consider the average degree in a network <k>What regime is the network in in the following cases? Explain what each regime means:

$$
\begin{gathered}
\langle\mathrm{k}\rangle<1 \\
\langle\mathrm{k}\rangle>1 \\
\langle\mathrm{k}\rangle\rangle \log N
\end{gathered}
$$

## TT08. Properties of rand. networks

What is the average distance between two nodes in an ER network of
N nodes and average degree <k>?

## TT08. Properties of rand. networks

What is the minimum average degree necessary for a graph to be connected?

## TT08. Properties of rand. networks

What is the expected clustering coefficient of a node in an ER network?

## TT09. Scale-free networks

## What does it mean to be scale-free in a scale-free network?

## TT09. Scale-free networks

Which probability distribution follows the degree of nodes in a scale-free network?

## TT09. Scale-free networks

Which networks do not exhibit a scale-free property?

## TT10. Distances in scale-free networks

What is the difference in the average distance of networks having $2<\gamma<3$ and networks having $\gamma>3$
?

# TT10. Distances in scale-free networks 

What is the friendship paradox?

## TT11 Preferential attachment

## Explain the Uniform Random Attachment model and its differences with the BA model

## TT11 Preferential attachment

What are the input parameters to the BA network model?

## TT11 Preferential attachment

Describe two of the steps requiring randomization of the preferential attachment generation algorithm

## TT12 Deg. preferential attachment

Which degree distribution have graphs generated using the BA model?

## TT12 Deg. preferential attachment

Which nodes have larger degree in a BA graph, those who are created early or those who are created late? Why?

## TT12 Deg. preferential attachment

What is the power-law exponent $\gamma$ of the degree distribution in a graph generated using the BA model?

Can this be changed within the BA model?

## TT12 Deg. preferential attachment

## Describe how to create a graph using the copy model

## TT14 Hubs and authorities

Execute some steps of HITS on this graph

| $\hat{H}(1)$ | $A(1)$ | $\hat{A}(1)$ | $H(2)$ | $\hat{\mathbf{H}}(\mathbf{2 )}$ | $A(2)$ | $\hat{A}(\mathbf{2 )}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |

## TT15 PageRank

Execute some steps of Simplified PageRank
underlined italics = normalized value

|  | $P(1)$ | $P(2)$ | $P(2)$ | $P(3)$ | $P(3)$ | $P(4)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 |  | $P(4)$ |  |  |  |
| 2 | 1 |  |  |  |  |  |
| 3 | 1 |  |  |  |  |  |
| 4 | 1 |  |  |  |  |  |
| 5 | 1 |  |  |  |  |  |
|  |  |  |  |  |  |  |

## TT15 PageRank

## Why do we use PageRank instead of Simplified PageRank?

What is the problem with Simplified PageRank?

## TT15 PageRank

In terms of the adjacency matrix of a graph, what is the PageRank of the nodes?

## TT17 Closeness

What is the closeness of a node?
What is the harmonic closeness of a node?

## TT17 Closeness

What is the closeness of one node in this graph?

## TT18 Betweenness

## What is the betweenness of a node?

## TT18 Betweenness

Why is the betweenness of the blue node on the left $12 ?$


## TT18 Betweenness

Compute betweenness using the Brandes-Newman algorithm


## TT18 Betweenness

Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*) should have betweenness approximately equal to N and closeness approximately $1 / \mathrm{N}$ for large N. Explain.

## TT19 Community structure

Give a example of a real-world network having two communities, and one having multiple communities

## TT20 Network flows

What is the max-flow problem?

## TT20 Network flows

What is the min-cut problem?

## TT10 Network flows

Why do we say max flow and min cut are equivalent problems?

## TT10 Network flows

Write the formulation of max flow as a linear system

## TT10 Network flows

Write the formulation of min cut as a linear system

## TT20 Network flows

Use the randomized algorithm we saw in class to find the min cut of this graph


## TT22 Dense sub-graphs

## Perform a k-core decomposition of this graph

## TT22 Dense sub-graphs

## Describe two density definitions that are commonly used

## TT22 Dense sub-graphs

What is the density definition used in Golderg's construction?

## TT22 Dense sub-graphs

## Draw Goldberg's construction on this graph for

 target density $5 / 2=2.5$
## TT22 Dense sub-graphs

Perform Charikar's algorithm on this graph; remember we measure density as |E|/|V|

## TT24 Spreading phenomena

Describe the linear threshold propagation model

## TT24 Spreading phenomena

## Describe the independent cascade propagation model

## TT24 Spreading phenomena

Run the linear threshold model on this graph starting from the node marked $*$


## TT24 Spreading phenomena

Run the independent cascade model on this graph starting from the node marked


## TT26 Epidemics

## Indicate what the basic reproductive number $R_{0}$ means

Indicate its formula in a branching process

## TT26 Epidemics

## Describe the SI model

What fraction of the nodes are infected at the end of a SI infection process?

## TT26 Epidemics

## Describe the SIS model

## Does the SIS model reach a steady state? How is this steady state called?

## TT26 Epidemics

## Describe the SIR model

What fraction of the nodes are recovered at the end of a SIR infection process?

## TT26 Epidemics

Describe the meaning of different variables in the following equations, which describe changes in the number of infected under a SIR process using conventional notation:
$\frac{d i(t)}{d t}=\beta\langle k\rangle i(t)(1-r(t)-i(t))-\mu i(t)$

## TT27 Epidemics on graphs

What is the characteristic time of an infection?

## TT27 Epidemics on graphs

What is the characteristic time as N grows in a scale-free network with $\gamma<3$ ?

