NAME	NIA	GRADE

Introduction to Network Science (2019-2020)

FINAL EXAM (TT01-TT13)

WRITE YOUR ANSWERS <u>CLEARLY</u> IN THE BLANK SPACES. Please write clearly, as if you were trying to communicate something to another person who needs to understand what you write to be able to evaluate it properly. If an answer requires intermediate steps, please mark clearly your final response with a rectangle. If you answer with text, please underline the key words or phrases of your answer. If absolutely necessary, you can attach an extra sheet to your exam, indicating that the solution can be found in the extra sheet.

Problem 1 *1 point*

Remembering the variance of the degree is $\sigma_k^2 = \left\langle k^2 \right\rangle - \left\langle k \right\rangle^2$, and that in a scale-free network, $\left\langle k^2 \right\rangle = C \frac{k_{\max}^{3-\gamma} - k_{\min}^{3-\gamma}}{3-\gamma}$, where C is a constant, $k_{\max}$ and $k_{\min}$ are the maximum and minimum degree, and γ is the power-law exponent.

1. Indicate what happens with the variance of the degree in a large graph when $\gamma \in]2, 3[$. Can you consider such a graph "scale-free"? Why or why not?

2. Indicate what happens with the variance of the degree in a large graph when $\gamma > 3$. Can you consider such a graph "scale-free"? Why or why not?

Problem 2 *1 point*

Given the following operations: G = *createGraph()* which creates en empty graph, *nodes(G)* which returns the nodes of the graph, *outLinks(G, i)* which returns the outlinks of node i, *addNode(G, i)* which adds a node numbered i to the graph, *addEdge(G, i, j)* which adds a directed edge from i to j to the graph, *pickRandom(S)* which picks a random element from set S. Describe a function implementing the copy model seen in class for creating a graph of n nodes using parameter p, in a precise manner with pseudocode.

runCopyModel(n,p):

Problem 3 *1 point*

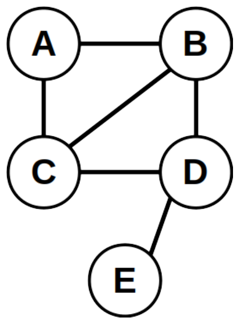
1. Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*), should have betweenness approximately equal to N and closeness approximately $1/N$ for large N. Explain briefly.

2. Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*), should have betweenness approximately equal to N and closeness approximately $2/N^2$ for large N. Explain briefly.

Do not use a concrete N. Use a general N, for instance by using the ellipsis (…) to denote multiple nodes.

Problem 4 *2 points*

Consider the graph on the right, which contains a subgraph with density $d(S) = 2|E(S,S)|/|S|$ equal to $5/2$. Draw the graph of Goldberg's construction, and in that graph, draw the $s-t$ cut that crosses some of the original edges and proves that a subgraph of density $5/2$ exists. Indicate clearly (1) the cost of each edge in the construction, (2) the desired target cost as a function of $|E|$, (3) the cost of the cut you found, and (4) the sub-graph the method finds.



1. Draw Goldberg's construction with costs of edges clearly indicated:

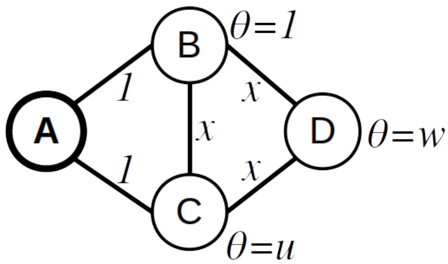
2. Cost of cut as a function of $|E|$ should be:

3. Cost of cut is:

4. Sub-graph with densith $5/2$ contains nodes:

Problem 5 *1 point*

Consider the graph on the right an the **Linear Threshold** model executed on it, starting from seed node A. The influence weights are written next to the edges, and the thresholds θ are written next to the nodes. Indicate what is the range of values of x for node C to be infected, but not node D. Justify briefly your answer.

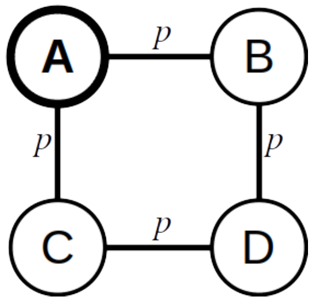


1. ____________ $\leq x <$ ____________
2. Justification:

Problem 6 *2 points*

Consider the graph on the right an the **Independent Cascade** model executed on it, starting from seed node A. The contagion probability of all edges is p.

Indicate what is the probability that at the end of the process:



1. Only node A is infected:
2. Only nodes A, B are infected:
3. Only nodes A, B, C are infected:
4. Only nodes A, B, D are infected:

Problem 7 *2 points*

In an epidemic model we will call **SIRS**, there are three possible states for a node: *susceptible*, *infected*, and *recovered*. Only susceptible nodes can become infected, only infected nodes can become recovered, and only recovered nodes can become susceptible again. During one unit of time, with probability β an infected node can infect one of its contacts, with probability μ, an infected node can recover, and with probability σ, a recovered node can become susceptible again.

Let $s(t)$ be the fraction of susceptible nodes, $i(t)$ be the fraction of infected nodes, $r(t)$ the fraction of recovered nodes, and $\langle k \rangle$ the average degree of the graph. Write the equations, **simplifying them appropriately**, for:

1. $\frac{di(t)}{dt} =$
2. $\frac{dr(t)}{dt} =$
3. $\frac{ds(t)}{dt} =$
4. Is $\sigma > \mu$ sufficient to say that the recovered will tend to zero in the long run?