| NAME | NIA | GRADE |
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# Introduction to Network Science (2019-2020)

*SECOND MID-TERM (TT05-TT09)*

**WRITE YOUR ANSWERS CLEARLY IN THE BLANK SPACES.** Please write clearly, as if you were trying to communicate something to another person who needs to understand what you write to be able to evaluate you properly. If an answer requires intermediate steps, please mark clearly your final response with a rectangle. If you answer with text, please underline the key words or phrases of your answer. If absolutely necessary, you can attach an extra sheet to your exam, indicating that the solution can be found in the extra sheet.

## Problem 1 — *2 point*

Consider a preferential attachment (BA) process in which nodes arrive one at a time, and each new node add $m$ links to the graph. Assume we currently have $N$ nodes numbered $1, 2, \ldots, N$. Currently the degree of the nodes is given by $k^{(N)}(i) = i$ (assume the graph may have self-loops). Now, node $N+1$ arrives and connects to the graph following the BA process.

1. What is the expected degree of node $i$ after node $N+1$ has connected to the rest of the graph, $k^{(N+1)}(i)$, as a function of $i, N$ and $m$?

2a. Is this increasing or decreasing on $i$?

2b. Is this increasing or decreasing on $N$?

## Problem 2 — *1 point*

In standard preferential attachment (BA), the probability that a new node connects to node $i$ is $\Pi(i) \propto k_i$ (proportional to the degree of node $i$).

In the following models, indicate **to which quantity is $\Pi(i)$ proportional to**. If there are extra parameters, indicate what do they mean and their range of possible values.

1. Super-linear preferential attachment.

2. Aging preferential attachment model, with a preference for old nodes.

**Problem 3** *3 points*

Consider a directed bi-partite graph $G = (V_L \cup V_R, E)$ in which $V_L = \{a, b, c, d\}$ and $V_R = \{1, 2, \ldots, 120\}$, and in which all edges go from a node in $V_L$ to a node in $V_R$:

- Node $a$ is connected to nodes $1, 2, \ldots 120$.
- Node $b$ is connected to nodes $1, 2, \ldots 60$.
- Node $c$ is connected to nodes $1, 2, \ldots 30$.
- Node $d$ is connected to nodes $1, 2, \ldots 15$.

Starting with $\hat{h}^{(1)}(i) = 1$ for $i \in \{a, b, c, d, 1, 2, \ldots, 120\}$.

1. Compute $a^{(1)}(i)$ for $i \in \{1, 2, \ldots, 120\}$

2. Compute $\hat{a}^{(1)}(i)$ for $i \in \{1, 2, \ldots, 120\}$

3. Compute $h^{(2)}(i)$ for $i \in \{a, b, c, d\}$

*Tip: if a set of nodes share the same value, just indicate the value and which nodes belong to that set.*

**Problem 4** *2 points*

Consider a directed graph $G = (V, E)$ in which $V = \{1, 2, \ldots, N\}$ and $(i, j) \in E \iff i \in V \wedge j \in V \wedge (j = i + 1 \vee j = i = N)$

1. Indicate the value of Simplified PageRank $S(i)$ for each node $i$ in the graph, justifying your answer.

2. Indicate the value of PageRank $P(i)$ for each node $i$ in the graph as a function of $i$ and the parameter $\alpha$.
   *Tip: write P(1), then write P(2), then write P(3), then write P(i).*

**Problem 5** *2 points*

Execute the Brandes-Newman algorithm for edge betweenness on the graph depicted on the right, indicating the intermediate values you computed. Copy your final answers to the edges of the graph depicted on the right.

Note: in graphs with cycles, **the edge betweenness can take fractional values**.