| NAME | NIA | GRADE |
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# Introduction to Network Science (2019-2020)

*FIRST MID-TERM (TT01-TT04)*

**WRITE YOUR ANSWERS CLEARLY IN THE BLANK SPACES.** Write as if you were trying to communicate something in written to another person who is going to evaluate what you write. If for some reason (for example, if after you have written the solution you realize that there is some mistake that you would like to correct) you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet. Also, you may use other sheets to perform your calculations.

## Problem 1 *1 point*

An analyst investigating reports of police corruption have created a graph in which each police officer is a node, and there is a link between two nodes if the officers have ever gone on patrol together. In this network, it is found that corrupt officers are often connected to each other. Indicate briefly what are the two possible explanations of this phenomenon.

1.

2.

## Problem 2 *1 point*

1. Write a formula for the average degree $\langle k \rangle$ of a network as a function of the number of nodes of the network, $N$, and the sum of the degree of nodes, $S$.

2. Write a formula for the expected number of links $L$ of an ER (Erdös-Renyi) graph of $N$ nodes that has expected average degree $\langle k \rangle$.

## Problem 3 *1 point*

Define briefly but **precisely**:

1. Connected graph

2. Clique

3. Bi-partite graph

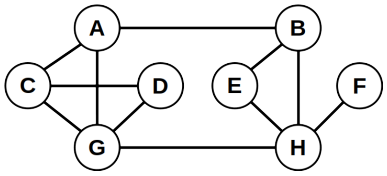
4. ER (Erdös-Renyi) graph

**Problem 4** *1 point*

Draw a connected and bi-partite directed graph of 10 nodes and 15 links in which there are no nodes with degree zero. Draw the degree distribution for this graph, indicating clearly the label of each axis. Write its adjacency matrix.

**Problem 5** *2 points*

Indicate the local clustering coefficient of each node in the figure.




- A.
- B.
- C.
- D.
- E.
- F.
- G.
- H.

**Problem 6** *1 point*

1. Indicate if an ER network of 192,244 nodes and 609,066 edges is in sub-critical, critical, super-critical, or connected regime, justifying briefly your answer.

2. Explain briefly what this means.

3. Indicate what is the minimum number of links this graph would need to be in connected regime.

4. Having $\langle k \rangle \geq 1$ is necessary, sufficient, or necessary and sufficient for an ER graph to be connected?

**Problem 7** *3 points*

Imagine a **connected** network of $N = 1,000,000$ nodes and average degree $\langle k \rangle = 5$ (indicate in each case the formula you are using and the result):

1. What is, approximately, the linking probability $p$ if this is an ER graph?

2. What is the average distance in this graph if the network is an ER graph?

3. What is the average distance in this graph if the network is a scale-free graph with high $\gamma$ exponent?

4. What is the average distance in this graph if the network is a scale-free graph with $\gamma \in ]2, 3[$?

5. If we draw at random 100 nodes from this network and it is scale-free, on expectation how many will have degree 1? (You can use the continuous approximation of the degree.)

6. If we draw at random 100 edges from this network and it is scale-free, and then randomly select one of the two nodes attached to this edge and discard the other node, on expectation how many of the remaining 100 nodes will have degree 1?