NAME	NIA	GRADE

Introduction to Network Science (2018-2019)

RECOVERY EXAM

WRITE YOUR ANSWERS <u>CLEARLY</u> IN THE BLANK SPACES. Write as if you were trying to communicate something in written to another person who is going to evaluate what you write. If for some reason (for example, if after you have written the solution you realize that there is some mistake that you would like to correct) you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet. Also, you may use other sheets to perform your calculations.

Problem 1 — *1 point*

Indicate the characteristics of an adjacency matrix representing:

1. A directed graph in which node i has out-degree r.

2. A directed graph in which node i has in-degree r.

3. A directed graph in which node i has a self-loop.

4. An undirected graph.

5. An undirected graph made of two connected components.

Problem 2 — *1 point*

In a random graph (Erdös-Rényi) of n nodes with connection probability p, what is

1. the expected number of links $\langle L \rangle$.

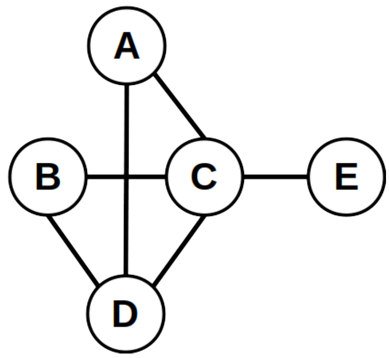
2. the average degree $\langle k \rangle$.

3. the mode (most frequent value) of the degree distribution.

4. the total number of nodes at distances $1, 2, \ldots, d$, i.e., the number of nodes at distance $\leq d$.

Problem 3 *1 point*

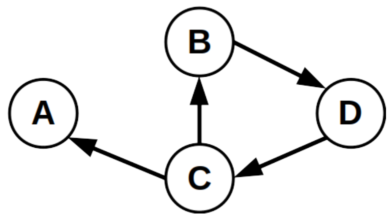
Indicate the following:



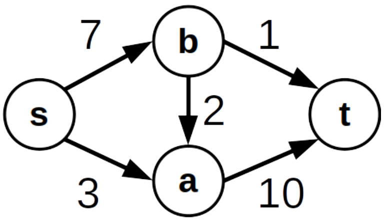
1. clustering coefficient of node A
2. clustering coefficient of node B
3. clustering coefficient of node C
4. clustering coefficient of node E
5. diameter of the graph

Problem 4 *2 point*

Compute two iterations of the hubs and authorities algorithm on this graph. Initialize all nodes to have a hub score of 1.0. Indicate all intermediate values of the hub and authority scores for every node.



Problem 5 *2 points*



1. Write the linear program ("minimize *objective function* subject to *constraints*") for minimum cut from s to t in this graph. Name the unknown variables $u_s, u_a, u_b, u_t, y_{sb}, y_{sa}, y_{bt}, y_{at}, y_{ab}$. Remember y_{ij} will be 1 if the edge from i to j is part of the minimum cut. Two constraints you must include are $u_s = 1$ and $u_t = 0$.

2. Guess the solution by visual inspection, indicate the value of all the u_i and y_{ij} variables in your solution.

3. Indicate the value of the objective function.

4. Show that each of the constraints is held.

Problem 6 *3 points*

Explain the following:

1. Why in the SI model, the derivative of the number of infected nodes $i(t)$ at time t is ...?

$$\frac{di(t)}{dt} = i(t) \langle k \rangle (1 - i(t))\beta$$

2. In the previous equation, what does the β parameter represent precisely?

3. Why in the SIS model, the derivative of the number of infected nodes $i(t)$ at time t is ...?

$$\frac{di(t)}{dt} = i(t) \langle k \rangle (1 - i(t))\beta - \mu i(t)$$

4. In the previous equation, what does the μ parameter represent precisely?

5. For the SIR model, write the equations for $\frac{di(t)}{dt}$, $\frac{dr(t)}{dt}$, $\frac{ds(t)}{dt}$, where at time t, $i(t)$ are the number of infected nodes, $r(t)$ the number of recovered nodes, and $s(t)$ the number of susceptible nodes.

6. In the SIR model, what is the limit of $i(t)$ when $t \to \infty$?