NAME	NIA	GRADE

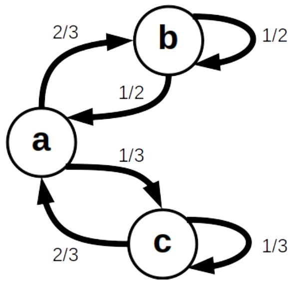
Introduction to Network Science (2018-2019)

PARTIAL EXAM 02

WRITE YOUR ANSWERS CLEARLY IN THE BLANK SPACES. Write as if you were trying to communicate something in written to another person who is going to evaluate what you write. If for some reason (for example, if after you have written the solution you realize that there is some mistake that you would like to correct) you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet. Also, you may use other sheets to perform your calculations.

Problem 1 *3 points*

Consider the graph on the right, and a weighted version of Simplified PageRank, in which the weights are provided next to the edges in the figure. These weights, which as you can see are normalized, indicate the proportion in which scores must be distributed through out-links. Denote by p_a, p_b, p_c the final scores of nodes a, b, and c respectively.



1. (2 points) Write a system of four equations, in which
   - the first three equations are the equilibrium equations for each node, indicating that the score of a node is the weighted sum of its incoming scores (or equivalently, that if we do one more iteration we obtain the same scores), and
   - the fourth equation indicates that p_a, p_b, p_c form a probability distribution.
2. (1 point) Solve this system of equations to determine p_a, p_b, p_c.

Problem 2 *3 points*

Consider a weighted version of the Hubs and Authorities algorithm. Let $G = (V, E, W)$ be a graph, $|V| = N$, $E \subseteq V \times V$, $W : E \to \mathbb{R}_0^+$. Let $f(i)$ denote the total out-weight of a node, $f(i) = \sum_{(i,j)\in E} W(i,j)$ and $b(i)$ the total in-weight of a node, $b(i) = \sum_{(j,i)\in E} W(j,i)$. For all nodes $i \in V$, we define the following:

$$h_0(i) = \widehat{h_0}(i) = \frac{1}{N}$$

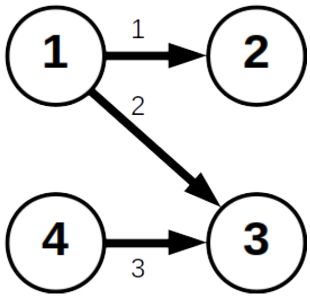
$$a_k(i) = \sum_{(j,i)\in E} h_k(j) \cdot \frac{W(j,i)}{f(j)} \qquad (k \geq 0) \qquad\qquad h_k(i) = \sum_{(i,j)\in E} a_{k-1}(j) \cdot \frac{W(i,j)}{b(j)} \qquad (k \geq 1)$$

$$\widehat{a_k}(i) = \frac{a_k(i)}{\sum_{j\in V} a_k(j)} \qquad\qquad \widehat{h_k}(i) = \frac{h_k(i)}{\sum_{j\in V} h_k(j)}$$

Define column vectors $\vec{h}_k$, $\vec{\widehat{h}}_k$, $\vec{a}_k$, $\vec{\widehat{a}}_k$ as follows:

$$\vec{h}_k = \begin{bmatrix} h_k(1) \\ h_k(2) \\ \vdots \\ h_k(N) \end{bmatrix} \qquad \vec{\widehat{h}}_k = \begin{bmatrix} \widehat{h_k}(1) \\ \widehat{h_k}(2) \\ \vdots \\ \widehat{h_k}(N) \end{bmatrix} \qquad \vec{a}_k = \begin{bmatrix} a_k(1) \\ a_k(2) \\ \vdots \\ a_k(N) \end{bmatrix} \qquad \vec{\widehat{a}}_k = \begin{bmatrix} \widehat{a_k}(1) \\ \widehat{a_k}(2) \\ \vdots \\ \widehat{a_k}(N) \end{bmatrix}$$

With these definitions, we can use square matrices R, S, of size $N \times N$ to write this computation as a matrix multiplication, $\vec{a}_k = R\vec{\widehat{h}}_k$, $\vec{h}_k = S\vec{\widehat{a}}_{k-1}$. For the graph on the right, where the numbers next to the edges are the edge weights W, provide numerically:

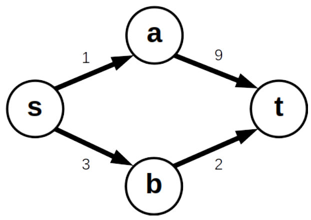


1. (1 point) The matrix R
2. (1 point) The vector $\vec{a_0}$ computed by matrix multiplication.
3. (1 point) The matrix S

Problem 3

1 point

Enumerate all possible (s,t)-cuts in the graph of the right, and indicate the cost of each one. Write each cut as a sub-set of edges from the set $\{(s,a),(s,b),(a,t),(b,t)\}$.

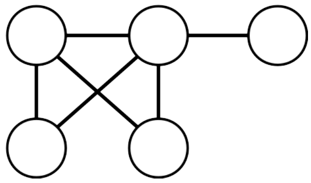


Problem 4

2 points

Consider the graph on the right.

1. (1 point) Indicate how many nodes are in each k-core of this graph.
2. (1 point) Draw a new graph, having the same nodes and just one additional edge, in which there are no nodes in the 2-core.



Problem 5

1 point

Provide a brief definition of the betweenness of an edge. You do not need to introduce notation to write this definition, as long as your definition is brief and unambiguous.