NAME	NIA	GRADE

Introduction to Network Science (2018-2019)

PARTIAL EXAM 01

WRITE YOUR ANSWERS CLEARLY IN THE BLANK SPACES. Write as if you were trying to communicate something in written to another person who is going to evaluate what you write. If for some reason (for example, if after you have written the solution you realize that there is some mistake that you would like to correct) you can attach an extra sheet to your exam. In this case, indicate clearly that the solution can be found in the extra sheet. Also, you may use other sheets to perform your calculations.

Problem 1

What is the minimum average distance attainable in a connected graph of N nodes? How? (Draw a sketch)

Problem 2

Consider the following adjacency matrix of a directed graph:

$$M = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

1. Draw the graph naming nodes A, B, C, and D

2. Is the graph connected, yes or no?

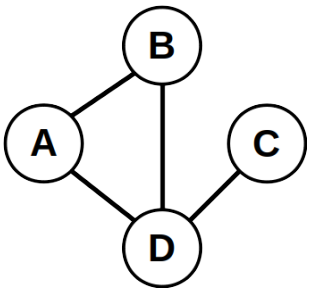
3. Write a table or draw a bar plot with the degree distributions (degree, probability):
   (3a) In-degree (3b) Out-degree

Problem 3

Consider a bipartite graph $G = (V, E)$ with $V = V_A \cup V_B$, $V_A \cap V_B = \emptyset$, $E \subseteq V_A \times V_B$. Suppose $\exists u \in V_A, k_u = |V_B|$, with k_u being the degree of u. Prove that the right-projection $G_B = (V_B, E_B)$ is a connected graph.

Problem 4

Consider the graph shown on the right. Indicate the clustering coefficent of:



1. Node A
2. Node B
3. Node C
4. Node D

Problem 5

In an ER network with N nodes and probability p of a link between the nodes, what is $\langle L \rangle$?

Problem 6

We consider an ER network is in the "connected regime" when $\langle k \rangle > \log N$. What does this mean?

Problem 7

Consider two ER networks with the following characteristics. Indicate if each one is in sub-critical, super-critical, or connected regime; justify briefly your answer.

1. An ER network with 1000 nodes and 5000 links

2. An ER network with 2000 nodes and 3000 links

Problem 8

For the same two graphs of the previous question, indicate the average distance between nodes; justify briefly your answer.

1. $\langle d \rangle =$

2. $\langle d \rangle =$

Problem 9

What does it mean that a network is scale-free?

Problem 10

Given a scale-free network of 1000 nodes and power-law coefficient $\gamma = 2.1$, what is the expected average distance between two nodes?